مختلط نمایی فوریه :

$$C_n = \frac{1}{2}(a_n - ib_n)$$

$$= \frac{1}{2}\left(\frac{\sin\alpha\pi}{\pi}\left(\frac{-2\alpha}{(4n^2-\alpha^2)}\right) - i\left(-\frac{1}{\pi}\right)\left(\frac{\cos\alpha\pi}{2n-\alpha} + \frac{\cos\alpha\pi}{2n+\alpha} - \frac{4n}{4n^2-\alpha^2}\right)\right)$$

$$F(x) = \sum_{n=-\infty}^{n=\infty} C_n e^{2nix}$$

دامنه مرکب فوریه :

$$\beta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$A_n = \sqrt{a_n^2 + b_n^2} \qquad F(x) = \frac{a_\circ}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{l}x - \beta_n\right)$$

ب) قضیه پارسوال :

$$\frac{2}{\pi}\int_0^{\pi} \cos^2 \alpha x dx = \frac{(2\sin\alpha\pi/\alpha\pi)^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2$$

$$\frac{2}{\pi}\int_0^{\pi} \frac{1+\cos 2\alpha x}{2} dx = \frac{2}{\pi}\left[\frac{1}{2}x + \frac{1}{4\alpha}\sin 2\alpha x\right] = \frac{2}{\pi}\left[\frac{1}{2}\pi + \frac{1}{4\alpha}\sin 2\alpha x\right]$$

$$1 + \frac{1}{2\alpha\pi}\sin\alpha x = \frac{2\sin^2\alpha\pi}{\alpha^2\pi^2} + \sum_{n=1}^{\infty}\frac{\sin^2\alpha\pi}{\pi^2}\left(\frac{4\alpha^2}{(4n^2-\alpha^2)^2}\right) + \frac{1}{\pi^2}\left(\frac{\cos\alpha\pi}{(2n-\alpha)} + \frac{\cos\alpha\pi}{(2n+\alpha)} - \frac{4n}{4n^2-\alpha^2}\right)^2$$

F.S (5

$F(x) = \sin\alpha x \qquad 0 < x < \pi \qquad \alpha \notin z \qquad l = \pi/2$

$$F(x) = \frac{a_0}{2} + \sum a_n \cos \frac{n\pi}{l} x + b_n \sin \frac{n\pi}{l} x$$

$$a_0 = \frac{1}{l} \int_c^{c+21} F(x)dx = \frac{2}{\pi} \int_0^{\pi} \sin \alpha x dx = \frac{2}{\pi} \left\{ -\frac{1}{\alpha} \cos \alpha x \right\}_0^{\pi}$$

$$= -\frac{2}{\alpha\pi} \{\cos \alpha\pi - 1\} \Rightarrow \frac{a_0}{2} = -\frac{1}{\alpha\pi} \{\cos \alpha\pi - 1\}$$

$$a_n = \frac{1}{l} \int_0^{c+21} F(x) \cos \frac{n\pi}{l} x dx = \frac{2}{\pi} \int_0^{\pi} \sin \alpha x \cos 2nx dx =$$

$$= \frac{1}{\pi} \int_0^{\pi} (\sin(\alpha + 2n)x + \sin(\alpha - 2n)x)dx =$$

$$= \frac{1}{\pi} \left\{ -\frac{1}{\alpha + 2n} \cos(\alpha + 2n)n \right\}_0^{\pi} + \frac{1}{\pi} \left\{ -\frac{1}{\alpha - 2n} \cos(\alpha - 2n)x \right\}_0^{\pi}$$

$$= \frac{1}{\pi} \left\{ -\frac{1}{\alpha + 2n} (\cos \alpha\pi - 1) \right\} + \frac{1}{\pi} \left\{ \frac{1}{\alpha - 2n} (\cos \alpha\pi - 1) \right\}$$

$$= -\frac{1}{\pi} (\cos \alpha\pi - 1) \left\{ \frac{1}{\alpha + 2n} + \frac{1}{\alpha - 2n} \right\} = \frac{2\alpha(\cos \alpha\pi - 1)}{-\pi(\alpha^2 - 4n^2)}$$

$$a_n = \frac{2\alpha(1 - \cos \alpha\pi)}{\pi(\alpha^2 - 4n^2)}$$

$$b_n = \frac{1}{l} \int_c^{c+21} F(x) \sin \frac{n\pi}{l} x dx = \frac{2}{\pi} \int_0^{\pi} \sin \alpha x . \sin 2nx dx$$

$$= -\frac{2}{\pi} \int_0^{\pi} (\cos(\alpha + 2n)x - \cos(x - 2n)x)dx$$

$$= -\frac{2}{\pi} \left\{ \frac{1}{\alpha + 2n} \sin(\alpha + 2n)x \right\}_0^{\pi} - \frac{2}{\pi} \left\{ -\frac{1}{\alpha - 2n} \sin(\alpha - 2n)x \right\}_0^{\pi}$$

$$= -\frac{2}{\pi} \left\{ \frac{1}{\alpha + 2n} \sin \alpha\pi \right\} - \frac{2}{\pi} \left\{ -\frac{1}{\alpha - 2n} \sin \alpha\pi \right\}$$

$$= -\frac{2}{\pi} \sin \alpha\pi \left\{ \frac{1}{\alpha + 2n} - \frac{1}{\alpha - 2n} \right\} = \frac{2}{\pi} \frac{4n}{\alpha - 4n} \sin \alpha\pi$$

$$b_n = \frac{8n \sin \alpha\pi}{\pi(\alpha^2 - 4n^2)}$$

F.S.S

$F(x) = \sin \alpha x$ $\quad -\pi < x < \pi$ $\quad \alpha \notin z$ $\quad l = \pi$

$$F(x)=\sum b_n \sin {}^{n\pi}\!/_{l}\, x$$

$$b_n = {}^{1}\!/_{\pi}\int_{-n}^{n}\sin\alpha x\sin nx dx = -{}^{1}\!/_{2\pi}\int_{-\pi}^{\pi}(\cos(\alpha+n)x-\cos(\alpha-n)x)dx$$

$$=-{}^{1}\!/_{\pi}\left\{{}^{1}\!/_{\alpha+n}\sin(\alpha+n)x\right\}_0^{\pi}+{}^{1}\!/_{\pi}\left\{{}^{1}\!/_{\alpha-n}\sin(\alpha-n)x\right\}_0^{\pi}$$

$$=-{}^{1}\!/_{\pi}\left\{\frac{(-1)^n}{\alpha+n}\sin\alpha\pi\right\}+{}^{1}\!/_{\pi}\left\{\frac{(-1)^n}{\alpha-n}\sin xdx\right\}$$

$$={}^{1}\!/_{\pi}\left((-1)^n\sin\alpha\pi\right)\left({}^{1}\!/_{\alpha-n}-{}^{1}\!/_{\alpha+n}\right)=\frac{2n\sin\alpha\pi(-1)}{\pi(\alpha^2-n^2)}$$

F.C.S

$$F(x)=\begin{cases}\sin\alpha x\\ -\sin\alpha x\end{cases}$$

$$a_n={}^{1}\!/_{l}\int_{c}^{c+21}F(x)\cos{}^{n\pi}\!/_{e}\,xdx=$$

$$={}^{1}\!/_{\pi}\int_{-\pi}^{0}-\sin\alpha x\cos xdx+{}^{1}\!/_{\pi}\int_{0}^{\pi}\sin\alpha x\cos nxdx$$

$$=-{}^{1}\!/_{2\pi}\int_{-x}^{0}(\sin(\alpha+n)x+\sin(\alpha-n)x)dn+{}^{1}\!/_{2\pi}\int_{0}^{\pi}(\sin(\alpha+n)x+\sin(\alpha-n)x)dx$$

$$=-{}^{1}\!/_{2\pi}\left\{-{}^{1}\!/_{\alpha+n}\cos(\alpha+n)n\right\}_{-\pi}^{0}-{}^{1}\!/_{2\pi}\left\{-{}^{1}\!/_{\alpha-n}\cos(\alpha-n)n\right\}_{-\pi}^{0}+{}^{1}\!/_{2\pi}\left\{-{}^{1}\!/_{\alpha+n}\cos(\alpha+n)x\right\}_0^{\pi}$$

$$+{}^{1}\!/_{2\pi}\left\{-{}^{1}\!/_{\alpha-n}\cos(\alpha-n)x\right\}_0^{\pi}$$

$$={}^{1}\!/_{2\pi}\left\{{}^{1}\!/_{\alpha+n}+{}^{1}\!/_{\alpha-n}-{}^{1}\!/_{\alpha+n}\cos(\alpha+n)\pi-{}^{1}\!/_{\alpha-n}\cos(\alpha-n)\pi\right\}+$$

$$+{}^{1}\!/_{2\pi}\left\{-{}^{1}\!/_{\alpha+n}\cos(\alpha+n)\pi-{}^{1}\!/_{\alpha-n}\cos(\alpha-n)\pi+{}^{1}\!/_{\alpha+n}+{}^{1}\!/_{\alpha-n}\right\}$$

$0<x<\pi \qquad \alpha\notin z \qquad l=\pi$

$-\pi<x<0$

$$a_n={}^{1}\!/_{\pi}\left\{{}^{1}\!/_{\alpha+n}+{}^{1}\!/_{\alpha-n}\right\}=\frac{2\alpha}{\pi(\alpha^2-n^2)}$$

سری مختلط نمايي:

$F(x)=\sin\alpha x \qquad 0<x<\pi \qquad \alpha\notin z$

$$F(x)=C_{\circ}+\sum c_n e^{in\pi/_{e}x} \qquad C_{\circ}=a_{0}/_{2}$$

$$C_n=\tfrac{1}{2}(a_n-ib_n)=\tfrac{1}{2}\left(\frac{2\alpha(1-\cos\alpha\pi)}{\pi(\alpha^2-4n^2)}-i\frac{8n(\sin\alpha\pi)}{\pi(\alpha^2-4n^2)}\right)$$

$$C_n=\frac{\alpha(1-\cos\alpha\pi)-4in(\sin\alpha x)}{\pi(\alpha^2-4n^2)}$$

دامنه مرکب فوریه:

$$F(x)=\sin\alpha x \qquad 0<x<\pi \qquad d\notin z$$

$$F(x)=A_{\circ}+\sum A_n\cos\left(n\pi/_{l}\,x-\beta_n\right) \qquad A_0=a_{0}/_{2}$$

$$tg\beta_n=b_n/_{a_n} \qquad \beta_n=tg^{-1}b_n/_{a_n} \qquad \beta_n=tg^{-1}\frac{4n\sin\alpha\pi}{\alpha(1-\cos\alpha\pi)}$$

$$A_n=\sqrt{a_n^2+b_n^2}$$

$$A_n=\sqrt{\frac{4\alpha^2(1-\cos\alpha\pi)^2+64n^2(\sin\alpha\pi)^2}{\pi^2(\alpha^2-4n^2)^2}}$$

قضیه دیریکله:

$$x=0\begin{cases}\lim F(x)=0\\ x\to 0^-\\ \lim F(x)=0\\ x\to 0^+\end{cases} \qquad \frac{0+0}{4}=0 \qquad F(0)=0$$

$$a_{0}/_{2}+\sum a\cos\frac{2n\pi}{x}x+b_n\sin\frac{2n\pi}{\pi}x$$

$$0=-1/_{\alpha\pi}\{\cos\alpha\pi-1\}+\sum\frac{2\alpha(1-\cos\alpha\pi)}{\pi(\alpha^2-4n^2)}$$

$$\sum\frac{2\alpha(1-\cos\alpha\pi)}{\pi(\alpha^2-4n^2)}=1/_{\alpha\pi}\{\cos\alpha\pi-1\}$$

$$\sum\frac{1}{\alpha^2-4n^2}=-1/_{2\alpha^2}$$

S.F (6

$$F(x) = |x| + \sinh x \qquad -1 < x < 1 \qquad l = 1$$

$$a_0 = \tfrac{1}{l}\int_{c}^{c+2l} F(x)dx = \int_{-1}^{1}\left(|x| + \sinh x\right)dx$$

$$= \int_{-1}^{0} -xdx + \int_{0}^{1} xdx + \int_{-1}^{1} \sinh xdx$$

$$= -x^2/2\Big|_{-1}^{0} + x^2/2\Big|_{0}^{1} + \cosh x\Big|_{-1}^{1}$$

$$= 1 + \tfrac{1}{2}\left(e + \tfrac{1}{e} - \tfrac{1}{e} - e\right) = 1 \qquad a_{0/2} = \tfrac{1}{2}$$

$$a_n = \int_{-1}^{1}\left(|x| + \sinh x\right)\cos n\pi x dx$$

$$= \int_{1}^{-1} |x| \cos n\pi x dx + \int_{-1}^{1} \sinh x \cos n\pi x dx$$

$$= \int_{-1}^{0} -x\cos n\pi x dx + \int_{0}^{1} x\cos n\pi x dx$$

$$= \left\{\left(\frac{-x}{n\pi}\sin n\pi x - \tfrac{1}{n^2\pi^2}\cos n\pi x\right)\right\}_{-1}^{0} + \left\{\left(\tfrac{1}{n\pi}x\sin n\pi x + \tfrac{1}{n^2\pi^2}\cos n\pi x\right)\right\}_{0}^{1}$$

$$= -\tfrac{1}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2} - \tfrac{1}{n^2\pi^2} \qquad a_n = \frac{2(-1)^n}{n^2\pi^2} - \frac{2}{n^2\pi^2}$$

$$a_n = \frac{2\left((-1)^n - 1\right)}{n^2\Pi^2}$$

$$b_n = \int_{-1}^{1}\left(|x| + \sinh x\right)\sin n\pi x =$$

$$= \int_{-1}^{1} |x| \sin n\pi x dx + \int_{-1}^{1} \sinh x \sin n\pi x dx \leftarrow$$ تابع زوج

فرد

$$= \int_{0}^{1} e^{x}\sin n\pi x dx - \int_{0}^{1} e^{-x}\sin n\pi x dx$$

I_1 | I_2

e^{x}	$\sin n\pi x$
e^{x}	$-\frac{1}{n\pi}\cos n\pi x$
e^{x}	$-\frac{1}{n^2\pi^2}\sin n\pi x$

$$I_1 = -\frac{1}{n\pi}e^{x}\cos n\pi x + \frac{1}{n^2\pi^2}I$$

$$I = \frac{1}{1-\frac{1}{n^2\pi^2}}\left(-\frac{1}{n\pi}e^{x}\cos n\pi x\right)$$

e^{-x}	$\sin n\pi x$
$-e^{-x}$	$-\frac{1}{n\pi}\cos n\pi x$
e^{x}	$-\frac{1}{n^2\pi^2}\sin n\pi x$

$$I = -\frac{1}{n\pi}e^{-x}\cos n\pi x - \frac{1}{n^2\pi^2}I$$

$$I = \frac{1}{1+n^2\pi^2}\left(-\frac{1}{n\pi}e^{-x}\cos n\pi x\right)$$

$$b_n = \frac{n^2\pi^2}{n^2\pi^2-1}\left\{-\frac{1}{n\pi}\left(e(-1)^n + e^{-1}(-1)^{n+1}\right)\right\} + \frac{n^2\pi^2}{n^2\pi^2+1}\left\{-\frac{1}{n\pi}\left(e^{-1}(-1)^n + e(-1)^{n+1}\right)\right\}$$

$$= \frac{-n\pi}{n^2\pi^2-1}\left\{(-1)^n(e-e^{-1})\right\} + \frac{n\pi}{n^2\pi^2+1}\left\{(-1)^n(e-e^{-1})\right\} = \left\{(-1)^{-1}(e-e\right\}\left\{\frac{n\pi}{n^2\pi^2+1} - \frac{n\pi}{n^2\pi^2-1}\right\}$$

F.C.S

$F(x) = |x| + \sinh x \quad -1 < x < 1$ $\quad F(x) = \begin{cases} x + \sinh x & 0 < x < 1 \\ x - 2 + dinh(x-2) & 1 < x < 2 \end{cases}$

$$F(x) = \begin{cases} x + \sinh x & 0 < x < 1 \\ -x - \sinh x & -1 < x < 0 \\ x - 2 + \sinh(x-2) & 1 < x < 2 \\ -x - 2 - \sinh(x+2) & -2 < x < -1 \end{cases}$$

$$F(x) = a + \sum a \cos {}^{n\pi}\!/_{e}\, x$$

$$a = {}^{1}\!/_{l} \int_{c}^{c+21} F(x) \cos {}^{n\pi}\!/_{e}\, xdx$$

$$= {}^{1}\!/_{2} \int_{-2}^{-1} (-x-2-\sinh(x+2)) \cos {}^{n\pi}\!/_{2}\, xdx + {}^{1}\!/_{2} \int_{-1}^{0} (-x - Sinhx) Cos \frac{n\pi}{2} xdx$$

$$+ \frac{1}{2} \int_{0}^{1} (x + Sinhx) Cos \frac{n\pi}{2} xdx + \frac{1}{2} \int_{1}^{2} (x-2+Sinh(x-2)) Cos \frac{n\pi}{2} xdx$$

$$= \frac{1}{2} \int_{-2}^{0} -xCos \frac{n\pi}{2} x - \frac{1}{2} \int_{-2}^{-1} 2Cos \frac{n\pi}{2} xdx - \frac{1}{2} \int_{-2}^{-1} Sinh(x+2) Cos \frac{n\pi}{2} xdx$$

$$- \frac{1}{2} \int_{-1}^{0} SinhxCos \frac{n\pi}{2} xdx + \frac{1}{2} \int_{0}^{2} xCos \frac{n\pi}{2} xdx + \frac{1}{2} \int_{0}^{1} SinhxCos \frac{n\pi}{2} xdx$$

$$+ \frac{1}{2} \int_{1}^{2} -2Cos \frac{n\pi}{2} xdx + \frac{1}{2} \int_{1}^{2} Sinh(x-2) Cos \frac{n\pi}{2} xdx =$$

$-x$	$Cos \frac{n\pi}{2} x$
-1	$\frac{2}{n\pi} Sin \frac{n\pi}{2} x$
0	$\frac{-4}{n^2\pi^2} Cos \frac{n\pi}{2} x$

e^{x+2}	$Cos \frac{n\pi}{2} x$
e^{x+2}	$\frac{2}{n\pi} Sin \frac{n\pi}{2} x$
e^{x+2}	$\frac{-4}{n^2\pi^2} Cos \frac{n\pi}{2} x$

e^{-x-2}	$Cos \frac{n\pi}{2} x$
$-e^{-x-2}$	$\frac{2}{n\pi} Sin \frac{n\pi}{2} x$
e^{-x-2}	$\frac{-4}{n^2\pi^2} Cos \frac{n\pi}{2} x$

e^{x}	$Cos \frac{n\pi}{2} x$
e^{x}	$\frac{2}{n\pi} Sin \frac{n\pi}{2} x$
e^{x}	$\frac{-4}{n^2\pi^2} Cos \frac{n\pi}{2} x$

$$= \left\{-\frac{2}{n^2\pi^2}\cos\frac{n\pi}{2}x\right\}_{-2}^{0} + \left\{\frac{2}{n\pi}\sin\frac{n\pi}{2}\right\} - \frac{1}{4}\left\{\frac{1}{1-\frac{4}{n^2\pi^2}}\frac{2}{n\pi}\left(e^{x+2}\sin\frac{n\pi}{2}x\right)\right\}_{-2}^{-1}$$

$$+ \frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{-x-2}\sin\frac{n\pi}{2}x\right\}_{-2}^{-1} - \frac{1}{4}\left\{\frac{1}{1-\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{x}\sin\frac{n\pi}{2}x\right\}_{-1}^{0}$$

$$+ \frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{-x}\sin\frac{n\pi}{2}x\right\}_{-1}^{0} + \left\{\frac{2}{n^2\pi^2}\cos\frac{n\pi}{2}x\right\}_{0}^{2} + \frac{1}{4}\left\{\frac{1}{1-\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{x}\sin\frac{n\pi}{2}x\right\}_{0}^{1} +$$

$$- \frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{-x}\sin\frac{n\pi}{2}x\right\}_{0}^{1} + \left\{\frac{2}{n\pi}\sin\frac{n\pi}{2}x\right\}_{1}^{2} + \frac{1}{4}\left\{\frac{1}{1-\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{x-2}\sin\frac{n\pi}{2}x\right\}_{1}^{2}$$

$$- \frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{2-x}\sin\frac{n\pi}{2}x\right\}_{1}^{2} = \frac{2}{n^2\pi^2}\left((-1)^n-1\right) + \frac{2}{n\pi}\sin\frac{n\pi}{2} +$$

$$+ \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2-4}\frac{2}{n\pi}e^{1}\sin\frac{n\pi}{2}\right\} - \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2+4}\frac{2}{n\pi}e^{-1}\sin\frac{n\pi}{2}\right\} - \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2-4}\frac{2}{n\pi}e^{-1}\sin\frac{n\pi}{2}\right\}$$

$$+ \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2}+4\frac{2}{n\pi}e^{1}\sin\frac{n\pi}{2}\right\} + \frac{2}{n^2\pi^2}\left((-1)^n-1\right) + \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2-4}\frac{2}{n\pi}e\sin\frac{n\pi}{2}\right\}$$

$$- \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2+4}\frac{2}{n\pi}e^{-1}\sin\frac{n\pi}{2}\right\} - \left\{\frac{2}{n\pi}\sin\frac{n\pi}{2}\right\} - \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2-4}\frac{2}{n\pi}e^{-1}\sin\frac{n\pi}{2}\right\}$$

$$+ \frac{1}{4}\left\{\frac{n^2\pi^2}{n^2\pi^2+4}\frac{2}{n\pi}e^{1}\sin\frac{n\pi}{2}\right\}$$

$$= \frac{4}{n^2\pi^2}\left((-1)^n-1\right) + \left\{\frac{n\pi}{n^2\pi^2}e\sin\frac{n\pi}{2}\right\} - \left\{\frac{n\pi}{n^2\pi^{2+4}}e^{-1}\sin\frac{n\pi}{2}\right\}$$

$$- \left\{\frac{n\pi}{n^2\pi^2-4}e^{-1}\sin\frac{n\pi}{2}\right\} + \left\{\frac{n\pi}{n^2\pi^2+4}e\sin\frac{n\pi}{2}\right\}$$

$$= \frac{4}{n^2\pi^2}\left((-1)^n - 1\right) + e\sin\frac{n\pi}{2}\left\{\frac{n\pi}{n^2\pi^2 - 4} + \frac{n\pi}{n^2\pi^2 + 4}\right\} - e^{-1}\sin {}^{n\pi}\!/\!_{2}\left\{\frac{n\pi}{n^2\pi^2 - 4} + \frac{n\pi}{n^2\pi^2 + 4}\right\}$$

$$a_n = \frac{4}{n^2\pi^2}\left((-1)^n - 1\right) + \left\{\frac{n\pi}{n^2\pi^2 - 4} + \frac{n\pi}{n^2\pi^2 + 4}\right\}\left\{e\sin {}^{n\pi}\!/\!_{2} - e^{-1}\sin\frac{n\pi}{2}\right\}$$

F.S.S (6

$$F(x) = \begin{cases} x + \sinh x & 0 < x < 1 \\ x - 2 + \sinh(x - 2) & 1 < x < 2 \end{cases}$$

$$F(x) = \begin{cases} x + \sinh x & -1 < x < 1 \\ x - 2 + \sinh(x - 2) & 1 < x < 2 \\ x + 2 + \sinh(x + 2) & -2 < x < -1 \end{cases} \qquad e = 2 \qquad \begin{matrix} F(-x) = -F(x) \\ x \to -x \\ F(x) \to -F(x) \end{matrix}$$

فـرد

$$F(x) = \sum b_n \sin {}^{n\pi}\!/\!_{e}\, x$$

$$b_n = {}^{1}\!/\!_{l} \int_{c}^{c+21} F(x) \sin {}^{n\pi}\!/\!_{e}\, x\,dx$$

$$= {}^{1}\!/\!_{2}\int_{-2}^{-1}\left(x + 2 + \sinh(x+2)\right)\sin {}^{n\pi}\!/\!_{2}\, x\,dx + {}^{1}\!/\!_{2}\int_{-1}^{1}\left(x + \sinh x\right)\sin {}^{n\pi}\!/\!_{2}\, x\,dx$$

$$+ {}^{1}\!/\!_{2}\int_{1}^{2}\left(x - 2 + \sinh(x-2)\right)\sin {}^{n\pi}\!/\!_{2}\, x\,dx =$$

$$= {}^{1}\!/\!_{2}\int_{-2}^{-1} x\sin {}^{n\pi}\!/\!_{2}\, x\,dx + \int_{-2}^{-1}\sin {}^{n\pi}\!/\!_{2}\, x\,dx + {}^{1}\!/\!_{2}\int_{-2}^{-1}\frac{e^{x+2} - e^{-x-2}}{2}\sin {}^{n\pi}\!/\!_{2}\, x\,dx$$

$$+ {}^{1}\!/\!_{2}\int_{-1}^{1} x\sin {}^{n\pi}\!/\!_{2}\, x + {}^{1}\!/\!_{2}\int_{-1}^{1}\sinh x \sin {}^{n\pi}\!/\!_{2}\, x + {}^{1}\!/\!_{2}\int_{1}^{2} x\sin {}^{n\pi}\!/\!_{2}\, x - \int_{1}^{2}\sin {}^{n\pi}\!/\!_{2}\, x\,dx$$

$$+ {}^{1}\!/\!_{2}\int_{1}^{2}\frac{e^{x-2} - e^{-x+2}}{2}\sin {}^{n\pi}\!/\!_{2}\, x\,dx = {}^{1}\!/\!_{2}\left\{\frac{-x}{{}^{n\pi}\!/\!_{2}}\cos {}^{n\pi}\!/\!_{2}\, x + {}^{4}\!/\!_{n\pi}\sin {}^{n\pi}\!/\!_{2}\, x\right\}_{-2}^{-1} +$$

$$+ \left\{-{}^{2}\!/\!_{n\pi}\cos {}^{n\pi}\!/\!_{2}\, x\right\}_{-2}^{-1} + {}^{1}\!/\!_{2}\left\{{}^{1}\!/\!_{2}\frac{-1}{1 - {}^{4}\!/\!_{n^2\pi^2}}\frac{2}{n\pi}e^{x+2}\cos {}^{n\pi}\!/\!_{2}\, x\right\}_{-2}^{-1} +$$

$$+\frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{2}{n\pi}e^{-x-2}\cos\frac{n\pi}{2}x\right\}_{-2}^{-1}+\frac{1}{2}\left\{\frac{-2x}{n\pi}\cos\frac{n\pi}{2}x+\frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}x\right\}_{-1}^{1}$$

$$+\frac{1}{4}\left\{\frac{1}{1-\frac{4}{n^2\pi^2}}\frac{-2}{n\pi}e\cos\frac{n\pi}{2}x\right\}_{-1}^{1}-\frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{-2}{n\pi}e^{-x}\cos\frac{n\pi}{2}x\right\}_{-1}^{1}+$$

$$+\frac{1}{2}\left\{\frac{-2x}{n\pi}\cos\frac{n\pi}{2}x+\frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}x\right\}_{1}^{2}+\left\{\frac{2}{n\pi}\cos\frac{n\pi}{2}x\right\}_{1}^{2}+$$

$$+\frac{1}{4}\left\{\frac{1}{1-\frac{4}{n^2\pi^2}}\frac{-2}{n\pi}e^{x-2}\cos\frac{n\pi}{2}x\right\}_{1}^{2}-\frac{1}{4}\left\{\frac{1}{1+\frac{4}{n^2\pi^2}}\frac{-2}{n\pi}e^{-x+2}\cos\frac{n\pi}{2}x\right\}_{1}^{2}$$

$$=\frac{1}{2}\left\{-\frac{2}{n\pi}x\cos\frac{n\pi}{2}x+\frac{4}{n\pi}\sin\frac{n\pi}{2}x\right\}_{-2}^{2}+2\left\{\frac{2}{n\pi}\cos\frac{n\pi}{2}x\right\}_{1}^{2}+$$

$$-\frac{1}{2}\left\{\frac{n\pi}{n\pi-4}e^{x+2}\cos\frac{n\pi}{2}x\right\}_{-2}^{-1}+\frac{1}{2}\left\{\frac{n\pi}{n^2\pi^2+4}e^{-x-2}\cos\frac{n\pi}{2}x\right\}_{-2}^{-1}$$