$$\left.\begin{aligned} y_{p2} &= A_1 \sin(2n-1)t \\ y'_{p2} &= A_1(2n-1)\cos(2n-1)t \\ y''_{p2} &= -A_1(2n-1)^2 \sin(2n-1)t \end{aligned}\right\} \Rightarrow -A_1(2n-1)^2 \sin(2n-1)t - A_1 \sin(2n-1)t = \frac{2}{(2n-1)\pi}\sin(2n-1)t$$

$$A_{(2n-1)} = \frac{-2}{\left((2n-1)^2+1\right)(2n-1)\pi}$$

$$y_{p2} = \sum_{n=1}^{\infty} A_{(2n-1)} \sin(2n-1)t$$

$$y = y_p + y_h = e_1 e^t + c_2 e^{-t} + \underbrace{\sum_{n=1}^{\infty} \frac{-2}{\left((2n-1)^2+1\right)(2n-1)\pi}\sin(2n-1)t}_{y_{p2}} \underbrace{- \tfrac{1}{2}}_{y_{p1}}$$

$$y(0) = 1 \rightarrow C_1 + C_2 = \tfrac{3}{2} \quad \underline{\underline{1}}$$

$$y'(0) = 0 \rightarrow y' = C_1 e^t - C_2 e^{-t} + \sum \frac{-2}{\left((2n-1)^2+1\right)(2n-1)\pi}\cos(2n-1)t$$

$$y'(0) = 0 \rightarrow C_1 - C_2 = \sum_{n=1} \frac{2}{\left((2n-1)^2+1\right)(2n-1)\pi} \quad \underline{\underline{2}}$$

از ضرایب <u>1</u> و <u>2</u> ضرایب بدست می آید.

---

قسمت ج سوال 37

$$F(t) = \begin{cases} 1 & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases} \qquad 2e = 2\pi \qquad y'' + y = F(t)$$

سری فوریه تابع $F(t)$ را می نویسیم.



$$a_0 = \frac{1}{\pi}\int_0^{\pi} dt = \frac{1}{\pi}(t)_0^{\pi} = 0$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \cos nt dt = \frac{1}{n\pi}\sin nt \Big|_0^{\pi} = 0$$

زوج $n \rightarrow 0$

$$b_n = \frac{1}{\pi}\int_0^{\pi} \sin n + dt = -\frac{1}{n\pi}\cos nt\Big|_0^{\pi} = \frac{1}{n\pi}\left((-1)^{n+1}+1\right) \rightarrow$$

فرد $n \rightarrow \frac{2}{n\pi}$

$$b_{2n-1} = \frac{2}{(2n-1)\pi}$$

$$y''+y = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)t \qquad y = y_h + y_p$$

$$y''+y = 0 \rightarrow t^2+1=0 \qquad t = ^{+}_{-} e \qquad y_h = C_1 \sin t + C_2 \cos t$$

$$y_{p1} = A \rightarrow 0 + A = \frac{1}{2} \rightarrow y_{p1} = \frac{1}{2}$$

به ازای $n=1$ داریم:

$$y_{p2} = t(A_1 \sin t + A_2 \cos t)$$
$$y'_{p2} = A_1 \sin t + A_2 \cos t + t(-A_2 \sin t + A_1 \cos t)$$
$$y''_{p2} = 2A_1 \cos t - 2A_2 \sin t - A_2 t \cos t - A_1 t \sin t$$
$$\rightarrow 2A_1 \cos t - 2A_2 \sin t - A_2 t \cos t - A_1 t \sin t + A_1 t \sin t + A_2 t \cos t$$
$$= \frac{2}{\pi}\sin t \Rightarrow \begin{cases} A_1 = 0 \\ -2A_2 = \frac{2}{\pi} \rightarrow A_2 = -\frac{1}{\pi} \end{cases}$$

$$y_{p2} = -\frac{t}{\pi}\cos t$$



به ازای بقیه $n$ ها داریم:

$$y_{p3} = A_1 \sin(2n-1)t + A_2 \cos(2n-1)t$$

$$y'_{p3} = (2n-1)A_1 \cos(2n-1)t \qquad y''_{p3} = -(2n-1)^2 A_1 (2n-1)t$$

$$\rightarrow -(2n-1)^2 A_1 \sin(2n-1)t + A_1 \sin(2n-1)t = \frac{2}{\pi(2n-1)}\sin(2n-1)t$$

$$\rightarrow A_1 = \frac{2}{\pi(2n-1)\left[1-(2n-1)^2\right]} \qquad y_{p3} = \frac{2}{\pi(2n-1)\left[1-(2n-1)^2\right]}$$

$$y = C_1 \sin t + C_2 \cos t + \frac{1}{2} - \frac{t}{\pi}\cos t + \sum_{n=2}^{\infty} \frac{2}{\pi(2n-1)\left[1-(2n-1)^2\right]} \sin(2n-1)t$$

$$y(0) = 1 \rightarrow 1 = C_2 + \frac{1}{2} \rightarrow C_2 = 1$$

$$y'(0) = 0 \rightarrow y' = C_1 \cos t - C_2 \sin t - \frac{\cos t}{\pi} + \frac{t}{\pi}\sin t + \sum_{n=2}^{\infty} \frac{2\cos(2n-1)t}{\pi\left[1-(2n-1)^2\right]}$$

$$0 = C_1 - \frac{1}{\pi} + \sum_{n=2}^{\infty} \frac{2}{\pi\left[1-(2n-1)^2\right]}$$

---

جواب سوال38

$$y'' - 3y' + 2y = F(t)$$

$$F(t) = \begin{cases} 1 & : 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases} \qquad l = \pi \leftarrow 2l = 2\pi$$

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} dx + \int_{\pi}^{2\pi} 0 dx = 1\right.$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} \cos nx dx + \int_{\pi}^{2\pi} (0)\cos nx dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\sin nx\Big|_0^{\pi}\right] = 0$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} \sin nx dx\right] = \frac{1}{\pi}\left[\frac{1}{n}\cos nx\Big|_0^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[(-1)^n - 1\right] \qquad \begin{cases} -\frac{2}{n\pi} & n = odd \\ 0 & n = even \end{cases}$$

$$F(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)\pi} \sin nx$$

$$y'' - 3y' + 2y = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1 \times \sin t}{\pi(2n-1)}$$

$$t^2 - 3t + 2 = 0$$

$$y_g = y_c + y_p \qquad y''-3y'+2y = 0 \qquad (t-1)(t-2) = 0 \to \begin{cases} t = 1 \\ t = 2 \end{cases}$$

$$y_c = C_1 e^t + C_2 e^{2t} \qquad y_p = K, y'' = y' = 0$$

$$2K = 1/2 \to K = 1/4 \Rightarrow y_{p1} = 1/4$$

در حالت دوم:

$$y''-3y'+2y = \sum_{n=1}^{\infty} \frac{1}{n\pi}\left(\left(-1\right)^n - 1\right) \times \sin nt$$

$$y_{p2} = A_1 \sin nt + A_2 \cos nt$$

$$y'_{p2} = nA_1 \cos nt - nA_2 \sin nt$$

$$y''_{p2} = -n^2 A_1 \sin nt - n^2 A_2 \cos nt$$

حال در معادله ی بالا جایگذاری می کنیم:

$$\left[-n^2 A_1 \sin nt - n^2 A_2 \cos nt + 3nA_1 \cos nt - 3nae \sin nt + 2A_1 \sin nt + 2A_2 \cos nt\right]$$

$$= \sum_{n=1}^{\infty} \frac{1}{n\pi}\left(\left(-1\right)^n - 1\right) \sin nt$$

ضرایب cos ها باید صفر شود.

$$-n^2 A_2 + 3nA + 2A_2 = 0$$

$$-n^2 A_1 + 3nA_2 + 2A_1 = \frac{1}{n\pi}\left(\left(-1\right)^n - 1\right)$$

جایگذاری در دومی

$$A_2(2 - n^2) + A_1(3n) = 0 \to A_2 = +A_1\left(\frac{3n}{n^2 - 2}\right)$$

$$-n^2 A_1 - 3n\left(A_1\left(\frac{3n}{n^2-2}\right)\right) + 2A_1 = \frac{1}{n\pi}\left(\left(-1\right)^n - 1\right)$$

$$A_1 = \left(-n^2 - \frac{n^2}{n^2 - 2} + 2\right) = \frac{1}{n\pi}\left(\left(-1\right)^n - 1\right)$$

$$A_1 = \frac{1}{n\pi}\left(\left(-1\right)^n - 1\right)\left(\frac{1}{2 - n^2 - n^2/\left(n^2 - 2\right)}\right)$$

$$A_2 = \left(3n \big/ n^2 - 2\right)(A_1)$$

$$y_p = y_{p1} + y_{p2}$$

و با لاخره $\longrightarrow$ $y = y_c + y_p$

39)

$F(t) = |t| \qquad -\pi < t < \pi \qquad F(t) = \begin{cases} t & 0 < x < \pi \\ -t & -\pi < x < 0 \end{cases}$

$y'' + 4y = F(t) \qquad y(0) = y'(0) = 0$

$$a_0 = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -t\,dt + \int_{0}^{\pi} t\,dt\right\} = \frac{1}{\pi}\left[\left\{-t^2\big/2\right\}_{-\pi}^{0} - \left\{t^2\big/2\right\}_{0}^{\pi}\right] = \frac{1}{\pi}\left(\pi^2\big/2 + \pi^2\big/2\right) = \pi$$

$$a_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -t\cos nt\,dt + \int_{0}^{\pi} t\cos nt\,dt\right\} = \frac{1}{\pi}\left\{\frac{-t}{n}\sin nt - \frac{1}{n^2}\cos nt\right]_{-\pi}^{0} + \frac{t}{n}\sin nt + \frac{1}{n^2}\cos nt\Big]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{-1}{n^2}\left(1 - (-1)^n\right) + \frac{1}{n^2}\left((-1)^n - 1\right)\right\} = \frac{2}{n^2\pi}\left((-1)^n - 1\right) = \begin{cases} 0 & n = even \\ \dfrac{-4}{\pi(2n-1)^2} & n = odd \end{cases}$$

$$\rightarrow a_n = \frac{-4}{\pi(2n-1)^n}$$

$$b_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -t\sin nt\,dt + \int_{0}^{\pi} t\sin nt\,dt\right\} = \frac{1}{\pi}\left[\left\{\frac{t}{n}\cos nt - \frac{1}{n}\sin nt\right\}_{-\pi}^{0}\right.$$

$$\left. + \left\{-\frac{t}{n}\cos nt + \frac{1}{n^2}\sin nt\right\}_{0}^{\pi}\right] = 0$$

برای تابع زوج در $b_n = 0$ است.

*بازه ی متقارن همواره



$$y'' + 4y = \pi\big/2 + \sum_{n=1}^{\infty}\left(\frac{-4}{\pi(2n-1)^2}\cos(2n-1)t\right)$$

$y_g = y_c + y_p$

$y_c = C_1 \sin 2t + C_2 \cos 2t$

$$y_{p1} = A \rightarrow y'_{p1} = 0 \rightarrow y''_{p1} = 0 \rightarrow 0 + 4A = \pi\big/2 \rightarrow A = \frac{\pi}{8}$$

$$y_{p2} = A_1 \sin(2n-1)t + A_2 \cos(2n-1)t \rightarrow A_1 = 0 \rightarrow y'_{p2} = -(2n-1)A_2 \sin(2n-1)]$$

$$\rightarrow y''_{p2} = -(2n-1)^2 A_2 \cos(2n-1)t$$

جایگذاری

$$-A_2(2n-1)^2\cos(2n-1)t+4A_2\cos(2n-1)t=\frac{-4}{\pi(2n-1)^2}\cos(2n-1)t$$

$$\to A_2\left(-(2n-1)^2+4\right)=-\frac{4}{\pi(2n-1)^2}\to A_2=\frac{-4}{\pi(2n-1)^2\left(4-(2n-1)^2\right)}$$

$$y_p=\frac{\pi}{8}+\sum_{n=1}^{\infty}\frac{-4}{\pi(2n-1)^2\left(4-(2n-1)^2\right)}\cos(2n-1)t$$

$$\to y_g=C_1\sin 2t+C_2\cos 2t+\frac{\pi}{8}+\sum_{n=1}^{\infty}\frac{-4}{\pi(2n-1)^2\left(4-(2n-1)^2\right)}\cos(2n-1)t$$

جایگذاری شرط ها:

$$y(0)=0\to C_2+\frac{\pi}{8}+\sum_{n=1}^{\infty}\frac{-4}{\pi(2n-1)^2\left(4-(2n-1)^2\right)}\cos(2n-1)t=0$$

$$\to C_2=-\frac{\pi}{8}-\sum_{n=1}^{\infty}\frac{-4}{\pi(2n-1)^2\left(4-(2n-1)^2\right)}$$

مشتق می گیریم از طرفین

$$y'_g=2C_1\cos 2t-2C_2\sin 2t+\sum_{n=1}^{\infty}\frac{-4\sin(2n-1)t}{\pi(2n-1)\left(4-(2n-1)^2\right)}$$

$$y'(0)=0\to 2C_1=0\to C_1=0$$

---



40)

$y''+9y=F(t),y(0)=1,y'(0)=\frac{1}{2}$ $\qquad F(t)=|t|,-\pi<\pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(t)dt = \frac{2}{\pi}\int_{0}^{\pi} tdt = \pi$$

$F(t)$ زوج تابع $\rightarrow b_n = 0 \rightarrow a_n = \frac{2}{\pi}\int_0^{\pi} t\cos ntdr$

زوج $\quad a_2 n = 0$

$$a_n = \frac{2}{\pi}\left(\frac{t}{n}\sin nx + \frac{1}{n^2}\cos nx\Big|_0^{\pi}\right) = \frac{2}{\pi n^2}\left((-1)^n - 1\right)$$ فرد $\rightarrow \quad a_{2n-1} = \frac{-4}{\pi(2n-1)^2}$

$$F(t) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{-4}{\pi(2n-1)^2}\cos nt$$

$y = y_h + y_p, y_h \rightarrow t^2 + 9 = 0 \rightarrow t = \pm 3i \Rightarrow y_h = A_1 \sin 3t + A_2 \cos 3t$

$y_{p1} \rightarrow 9y_{p1} = \frac{\pi}{2} \rightarrow y_{p1} = \frac{\pi}{18}$

$y_{p2} \rightarrow B_1 \sin nt + B_2 \cos nt$ (جایگذاری در رابطه)

$\Rightarrow y'' + 9y = \sum a_{2n-1}\cos nt$

$-n^2 B_1 \sin nt + 9B_2 \cos nt = \sum a_{2n-1}\cos nt$

$\rightarrow$ باید ضریب $\sin nt$، صفر شود. $\Rightarrow n^2 B_1 = 0 \rightarrow B_1 = 0$

$$9B_2 = \frac{-4}{\pi(2n-1)^2} \rightarrow B_2 = \frac{-4}{9\pi(2n-1)^2}$$

$y(0) = 1 \rightarrow y(x) = y_h(x) + y_p(x) \xrightarrow{x=t} y(0) = A_2 + \frac{\pi}{18} - \frac{4}{9\pi(2n-1)^2} = 1$

$y'(0) = \frac{1}{2} \rightarrow 3A_1 + 0 = \frac{1}{2} \rightarrow A_1 = \frac{1}{6}$

$$A_2 = 1 - \frac{\pi}{18} + \frac{4}{9\pi(2n-1)^2}$$



---

41)

$F(t) = |t| \quad , -\pi < t < \pi \quad , y'' + 4y' + 4y = F(t) \quad , y(0) = 1 \quad , y'(0) = 0$

$F(t)$ فوریه سری $\rightarrow 2l = 2\pi \qquad , l = \pi$

$$a_0 = \frac{1}{l}\int_c^{c+21} F(t)dt = \frac{1}{\pi}\int_{-\pi}^{\pi}|t|dt = \frac{2}{\pi}\int_0^{\pi} tdt = \pi$$

$$a_n = \frac{1}{l}\int_c^{c+21} F(t)\cos\frac{n\pi}{l}tdt = \frac{1}{\pi}\int_{-\pi}^{\pi}|t|\cos tdt = \frac{2}{\pi}\int_0^{\pi} t\cos ntdt$$

| | |
|---|---|
| $(t)t$ | $\cos nt$ |
| $(-)1$ | $\frac{1}{n}\sin nt$ |
| $(+)0$ | $-\frac{1}{n^2}\cos nt$ |

$$= \frac{2}{\pi}\left(\frac{t}{n}\sin nt + \frac{1}{n^2}\cos nt\right)_0^{\pi} = \frac{2}{\pi n^2}\left((-1)^n - 1\right)$$

تابع زوج است $\rightarrow b_n = 0$

$$F(t) = y''+4y'+4y = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}t + b_n \sin\frac{n\pi}{l}t = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left((-1)^n - 1\right)\cos nt$$

---

$y_g = y_c + y_p \qquad , y''+4y'+4y = 0 \rightarrow t^2 + 4t + 4 = 0 \rightarrow t_1 = t_2 = -2$

$\rightarrow y_c = C_1 e^{-2t} + C_2 te^{-2t}$

جوابهای خصوصی $\rightarrow y''+4y'+4y = \frac{\pi}{2} \rightarrow \left.\begin{matrix} y_{p1} = k \\ y'_{p1} = 0 \\ y''_{p1} = 0 \end{matrix}\right\} 4k = \frac{\pi}{2} \rightarrow k = \frac{\pi}{8} \rightarrow y_{p1} = \frac{\pi}{8}$

$$y''+4y'+4y = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left((-1)^n - 1\right)}{n^2}\cos nt \;(\mathrm{I}) \qquad , y_{p2} = [A_1 \sin nt + A_2 \cos nt]$$

$y'_{p2} = nA_1 \cos nt - nA_2 \sin nt \qquad , y''_{p2} = -n^2 A_1 \sin nt - n^2 A_2 \cos nt$ $(\mathrm{I})$ جاگذاری

در معادله

$$-n^2A_1\sin nt-n^2A_2\cos nt+4nA_1\cos nt-4nA_2\sin nt+4A_1\sin nt+4A_2\cos nt=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)}{n^2}\cos nt$$

$$\Rightarrow(-n^2A_1-4nA_2+4A_1)=0\rightarrow A_1(4-n^2)=4nA_2\rightarrow A_1=\frac{4n}{4-n^2}A_2$$

$$4A_2+4nA_1-n^2A_2=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)}{n^2}\Rightarrow 4A_2+\frac{16n}{4-n^2}A_2-n^2A_2=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)}{n^2}$$

$$\Rightarrow A_2\left(\frac{\left(n^2+4\right)^2}{(4-n^2)}\right)=\frac{2}{\pi}\left(\frac{(-1)^n-1}{n^2}\right)\Rightarrow A_2=\frac{2(4-n^2)\left((-1)^n-1\right)}{\pi n^2(n^2+4)^2}\rightarrow A_1=\frac{8\left((-1)^n-1\right)}{\pi n\left(n^2+4\right)^2}$$

$$y_{p2}=\sum_{n=1}^{\infty}\frac{8\left((-1)^n-1\right)}{\pi n(n^2+4)^2}\sin nt+\frac{2(4-n^2)\left((-1)^n-1\right)}{\pi n^2(n^2+4)^2}\cos nt$$

$$y_p=y_{p1}+y_{p2}=\frac{\pi}{8}+y_{p2}$$

$$y_g=y_c+y_p=C_1e^{-2t}+C_2te^{-2t}+y_p \quad \text{(II)}$$

از این معادله $C_1$ بدست می آید.

$y(0)=1$

$y'(0)=0$

$$\rightarrow y(0)=C_1+\frac{\pi}{8}+\sum_{n=1}^{\infty}\frac{2(4-n^2)\left((-1)^n-1\right)}{\pi n^2(n^2+4)^2}=1$$

$$\rightarrow y'(t)=-2c_1e^{-2t}+c_2e^{-2t}-2c_2te^{-2t}+\sum_{n=1}^{\infty}\frac{8}{\pi}\frac{\left((-1)^n-1\right)}{(n^2+4)^2}\cos nt+$$

$$\frac{-2(4-n^2)}{\pi n}\frac{\left((-1)^n-1\right)}{(n^2+4)^2}\sin nt \qquad \rightarrow y'(0)=0$$

با پیدا کردن $C_1$ از معادله بالا $C_2$ نیز بدست می آید.

$$0=-2C_1+C_2+\sum_{n=1}^{\infty}\frac{8}{\pi}\frac{\left((-1)^n-1\right)}{(n^2+4)^2}$$



پس از پیدا کردن $C_2,C_1$ مقادیر آنها را در معادله (II)جاگذاری می کنیم.

---

42)

$$y''+6y'+8y=F(t) \qquad ,y(0)=y'(0)=0 \qquad ,F(t)=|t| \qquad ,-\pi<t<\pi$$

مانند سوال قبلی: $F(t)=y''-6y'+8y=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left((-1)^n-1\right)\cos nt$

$$y_g=y_c+y_p \qquad ,y''+6y'+8y=0\rightarrow t^2+6t+8=0\rightarrow (t+4)(t+2)=0$$

$$\Rightarrow t_1=-4,t_2=-2\rightarrow y_c=C_1e^{-4t}+C_2e^{-2t}$$

جوابهای خصوصی: $\Rightarrow y''+6y'+8y=\frac{\pi}{2}$

$$\Rightarrow \left.\begin{array}{l} y_{p1}=k \\ y'=0 \\ y''=0 \end{array}\right\}\Rightarrow 8k=\frac{\pi}{2}\rightarrow k=\frac{\pi}{16}\Rightarrow y_{p1}=\frac{\pi}{16}$$

$$y''+6y'+8y=\sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left((-1)^n-1\right)\cos nt \ \text{(I)}$$

$$y_{p2}=\left[A_1\sin nt+B_2\cos nt\right]\rightarrow y'_{p2}=nA_1\cos nt-nA_2\sin nt$$

$y''_{p2}=-n^2A_1\sin nt-n^2A_2\cos nt$ $\xrightarrow{\text{جاگذاری در معادله (I)}}$

$$-n^2A_1\sin nt-n^2A_2\cos nt+6nA_1\cos nt-6nA_2\sin nt+8A_1\sin nt+8A_2\cos nt=$$

$$\sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left((-1)^n-1\right)\cos nt\rightarrow \sin nt\left(-n^2A_1-6nA_2+8A_1\right)=0\rightarrow$$

$$-n2A_1-6nA_2+8A_1=0\rightarrow A_1(8-n^2)=6nA_2\rightarrow A_1=\frac{6n}{8-n^2}A_2$$

$$-n^2A_2+6nA_1+8A_2=\sum_{n=1}^{\infty}\frac{2}{\pi n^2}((-1)^n-1)\Rightarrow$$

$$-n^2A_2+\frac{36n^2}{8-n^2}A_2+8A_2=\frac{2}{\pi n^2}((-1)^n-1)\Rightarrow A_2=\frac{2(8-n^2)((-1)^n-1)}{\pi n^2(n^4+20n^2+64)}$$

$$\Rightarrow A_1=\frac{12}{\pi n}\frac{\left((-1)^n-1\right)}{(n^4+20n^2+64)}$$

$$y_{p2}=\sum_{n=1}^{\infty}\frac{12}{\pi n}\frac{\left((-1)^n-1\right)}{(n^4+20n^2+64)}\sin nt+\frac{2(8-n^2)((-1)^n-1)}{\pi n^2(n^4+20n^2+64)}\cos nt$$