$$\frac{1}{4}\left(e^{x-y}-e^{y-x}+e^{x+y}-e^{-(x+y)}\right)$$

$$F(x,y)=\frac{a_0(y)}{2}+\sum_{m=1}^{\infty}a_m(y)\cos\frac{m\pi}{a}x+b_m(y)\sin\frac{m\pi}{a}n$$

$$F(x,y)=\frac{a_0(y)}{2}+\sum_{m=1}^{\infty}a_m(y)\cos 2m\pi x+b_m(y)\sin 2m\pi x$$

$$a_0(y)=2\int_0^1\frac{1}{4}\left(e^{x-y}-e^{y-x}+e^{x+y}-e^{-(x+y)}\right)dx=\frac{1}{2}\left(e^{x+y}+e^{-(x+y)}+e^{x-y}+e^{x-y}\right)=\frac{1}{2}\left[\left(e^{1-y}+e^{y-1}+e^{1+y}+e^{-1-y}\right)\right.$$

$$\left.-\left(e^{-y}+e^{y}+e^{y}+e^{-y}\right)\right)=\frac{1}{2}\left[e^{1-y}+e^{y-1}+e^{1+y}+e^{-1-y}-2e^{-y}-2e^{y}\right]$$

$$a_m(y)=2\int_0^1\frac{1}{4}\left(e^{x-y}-e^{y-x}+e^{x+y}-e^{-(x+y)}\right)\cos 2m\pi x\,dx\rightarrow$$

$$I_1=\int_0^1\frac{e^{x-y}}{2}\cos 2m\pi x\,dx=\frac{e^{-y}}{2}\int_0^1 e\cos 2m\pi x\,dx$$

$(+)e^{x}$	$\cos 2m\pi x$
$(-)e^{x}$	$\frac{1}{2m\pi}\sin 2m\pi x$
$(+)e^{x}$	$-\frac{1}{4m^2\pi^2}\cos 2m\pi x$
	$\int -\frac{1}{4m^2\pi^2}\cos \pi x$

$$=\frac{e^{-y}}{2}\left(\frac{e^x}{2m\pi}\sin 2m\pi x+\frac{e^x}{4m^2\pi^2}\cos 2m\pi x-\int_0^1\frac{e^x}{4m^2\pi^2}\cos m\pi x\,dx\right)\Bigg|_0^1\rightarrow$$

$$\left(1+\frac{1}{4m^2\pi^2}\right)\int_0^1 e^x\cos 2m\pi x\,dx=\left(\frac{e^x}{2m\pi}\sin 2m\pi x+\frac{e^x}{4m^2\pi^2}\cos 2n\pi x\right)\Bigg|_0^1\rightarrow$$

$$\int_0^1 e^x\cos 2m\pi x\,dx=\frac{e-1}{4m^2\pi^2+1}\rightarrow I_1=\frac{e^{-y}}{2}\left(\frac{e-1}{4m^2\pi^2+1}\right)$$

$$I_2=\int_0^1-\frac{e^{y-x}}{2}\cos 2m\pi x\,dx=\frac{e^{y}}{2}\int_0^1-e^{-x}\cos 2m\pi x\,dx=$$

$(+)-e^{-x}$	$\cos 2m\pi x$
$(-)e^{-x}$	$\frac{1}{2m\pi}\sin 2m\pi x$
$(+)-e^{-x}$	$-\frac{1}{4m^2\pi^2}\cos m\pi x$
	$\int -\frac{1}{4m^2\pi^2}\cos 2m\pi x$

$$=\frac{e^{y}}{2}\left(\frac{e^{-x}}{2m\pi}\sin 2m\pi x+\frac{e^{-x}}{4m^2\pi^2}\cos 2m\pi x-\int_0^1\frac{e^{-x}}{4m^2\pi^2}\cos m\pi x\,dx\right)\Bigg|_0^1$$

$$\Rightarrow\left(1+\frac{1}{4m^2\pi^2}\right)\int_0^1-e^{-x}\cos 2m\pi x\,dx=\left(\frac{e^{-x}}{2m^2\pi^2}\cos 2n\pi x\right)\Bigg|_0^1$$

$$\Rightarrow\int_0^1-e^{-x}\cos 2m\pi x\,dx=\frac{e^{-1}-1}{4m^2\pi^2+1}\rightarrow I_2=\frac{e^{y}}{2}\left(\frac{e^{-1}-1}{4m^2\pi^2+1}\right)$$

با توجه به I_1 $\rightarrow I_3=\int_0^1 \frac{e^{x+y}}{2}\cos 2m\pi x dx=\frac{e^y}{2}\left(\frac{e-1}{4m^2\pi^2+1}\right)$

با توجه به I_2 $\rightarrow I_4=\int_0^1 \frac{-1}{2}e^{-(x+y)}\cos 2m\pi x dx=\frac{e^{-y}}{2}\left(\frac{e^{-1}-1}{4m^2\pi^2+1}\right)$

$$\Rightarrow a_m(y)=I_1+I_2+I_3+I_4=\frac{e^{1-y}-e^{-y}+e^{y-1}-e^y+e^{y+1}-e^y+e^{-y-1}-e^{-y}}{2\left(4m^2\pi^2+1\right)}=$$

$$\frac{e^{1-y}-2e^{-y}+e^{y-1}-2e^y+e^{y+1}+e^{-y-1}}{2(4m^2\pi^2+1)}$$

$$b_m(y)=2\int_0^1 \frac{1}{4}\left(e^{x-y}-e^{y-x}+e^{x+y}-e^{-(x+y)}\right)\sin 2m\pi x dx \Rightarrow$$

$$I_1=\int_0^1 \frac{e^{x-y}}{2}\sin 2m\pi x dx=\frac{e^{-y}}{2}\int_0^1 e^x \sin 2m\pi x dx \Rightarrow$$

$(+)e^x$	$\sin 2m\pi x$
	$-\frac{1}{2}\cos 2m\pi x$
$(-)e^x$	$-\frac{1}{4m^2\pi^2}\sin 2m\pi x$
$(+)e^x$	$\int -\frac{1}{4m^2\pi^2}\sin 2m\pi x dx$

$$=\frac{e^{-y}}{2}\left(\frac{-e^x}{2m\pi}\cos 2m\pi x+\frac{e^x}{4m^2\pi^2}\sin 2n\pi x-\int_0^1 \frac{e^x}{4m^2\pi^2}\sin 2m\pi x dx\right)_0^1$$

$$\Rightarrow\left(1+\frac{1}{4m^2\pi^2}\right)\int_0^1 e^x \sin 2m\pi x dx=\frac{1-e}{2m\pi}\rightarrow$$

$$I_1=\frac{e^{-y}m\pi(1-e)}{4m^2\pi^2+1}$$

$$I_2=\int_0^1 -\frac{e^{y-x}}{2}\sin 2m\pi x dx=\frac{e^y}{2}\int_0^1 -e^{-x}\sin 2m\pi x dx=$$

$(+)-e^{-x}$	$\sin 2m\pi x$
	$-\dfrac{1}{2m\pi}\cos 2m\pi x$
$(-)e^{-x}$	$-\dfrac{1}{4m^2\pi^2}\sin 2m\pi x$
$(+)-e^{-x}$	$\int \dfrac{-1}{4m^2\pi^2}\sin 2m\pi x$

$$=\frac{e^y}{2}\left(\frac{-e^{-x}}{2m\pi}\cos 2m\pi x+\frac{e^{-x}}{4m^2\pi^2}\sin 2n\pi x-\int_0^1\frac{e^{-x}\sin 2m\pi x}{4m^2\pi^2}\right)\Bigg|_0^1$$

$$\Rightarrow\left(1+\frac{1}{4m^2\pi^2}\right)\int_0^1 e^{-x}\sin 2m\pi x dx=\frac{e^{-1}-1}{2m\pi}\rightarrow$$

$$I_2=\frac{e^y\left(\left(e^{-1}-1\right)(m\pi)\right)}{4m^2\pi^2+1}$$

با توجه به I_1 $\rightarrow I_3=\displaystyle\int_0^1-\frac{e^{y+x}}{2}\sin 2m\pi x dx=\frac{e^y m\pi(1-e)}{4m^2\pi^2+1}$

با توجه به I_2 $\rightarrow I_4=\displaystyle\int_0^1-\frac{-e^{-(x+y)}}{2}\sin 2m\pi x dx=\frac{e^{-y}(e^{-1}-1)m\pi}{4m^2\pi^2+1}$

$$b_m(y)=I_1+I_2+I_3+I_4=\frac{m\pi(e^{-y}-e^{1-y}+e^{y-1}-e^y+e^y-e^{y+1}+e^{-y-1}-e^{-y})}{4m^2\pi^2+1}=$$

$$\frac{m\pi(e^{y-1}-e^{1-y}-e^{y+1}+e^{-y-1})}{4m^2\pi^2+1}$$

$$F(x,y)=\frac{a_0(y)}{2}+\sum_{m=1}^{\infty}a_m(y)\cos 2m\pi x+b_m(y)\sin 2m\pi x \quad (\mathrm{I})$$

$$C^y=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos 2n\pi y+b_n\sin 2n\pi y)$$

$$a_0=2\int_0^1 e^y dy=2(e-1)$$

$$a_n=2\int_0^1 e^y\cos 2n\pi y dy=2\left(\frac{e^y}{2n\pi}\sin 2n\pi y+\frac{e^y}{4n^2\pi^2}\cos 2n\pi y-\int_0^1\frac{e^y}{4n^2\pi^2}\cos 2n\pi y dy\right)\Bigg|_0^1$$

	$\cos 2n\pi x$
$+e^y$	$\dfrac{1}{2n\pi}\sin 2n\pi y$
$-e^y$	$-\dfrac{1}{4n^2\pi^2}\cos 2n\pi y$
$+e^y$	$\int -\dfrac{1}{4n^2\pi^2}\cos 2n\pi y$

$$\Rightarrow\left(1+\frac{1}{4n^2\pi^2}\right)\int_0^1 e^y\cos 2n\pi y dy=\frac{e-1}{4n^2\pi^2}$$

$$\rightarrow a_n=\frac{2(e-1)}{4n^2\pi^2+1}$$

$$b_n = 2\int_0^1 e^y \sin 2n\pi y dy = 2\left(\frac{-e^{-y}}{2n\pi}\cos 2n\pi y + \frac{e^y}{4n^2\pi^2}\sin 2n\pi y - \int_0^1 \frac{e^y}{4n^2\pi^2}\sin 2n\pi y dy\right)_0^1$$

	$\sin 2n\pi x$
$+e^y$	$-\frac{1}{2m\pi}\cos 2n\pi x$
$-e^y$	$-\frac{1}{4n^2\pi^2}\sin 2n\pi y$
$+e^y$	$\int \frac{-1}{4n^2\pi^2}\sin 2n\pi y$

$$\Rightarrow \left(1+\frac{1}{4n^2\pi^2}\right)\int_0^1 e^y \sin\ 2n\pi y dy = \frac{1-e}{2n\pi}$$

$$b_n = \frac{(1-e)(4n\pi)}{4n^2\pi^2+1}$$

$$e^y = (e-1) + \sum_{n=1}^{\infty}\left(\frac{2(e-1)}{4n^2\pi^2+1}\cos 2n\pi y + \frac{(1-e)(4n\pi)}{4n^2\pi^2+1}\sin 2n\pi y\right)\text{(II)}$$

$$C^{-y} = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos 2n\pi y + b_n \sin 2n\pi y\right)$$

$$a_0 = 2\int_0^1 e^{-y}dy = 2(-e^{-y})_0^1 = 2(1-e^{-1})$$

$$a_n = 2\int_0^1 e^{-y}\cos 2n\pi y dy = 2\left(\frac{e^{-y}}{2n\pi}\sin 2n\pi y - \frac{e^{-y}}{4n^2\pi^2}\cos 2n\pi y - \int_0^1 \frac{e^{-y}}{4n^2\pi^2}\cos 2n\pi y dy\right)_0^1$$

$+e^{-y}$	
$-e^{-y}$	
$+e^{-y}$	

$$\Rightarrow \left(1+\frac{1}{4n^2\pi^2}\right)\int_0^1 e^{-y}\cos 2n\pi y dy = \frac{(-e^{-1}+1)}{4n^2\pi^2} \Rightarrow a_n = \frac{2(1-e^{-1})}{4n^2\pi^2+1}$$

با توجه به a_n $\rightarrow b_n = \frac{(1-e^{-1})4n\pi}{4n^2\pi^2+1}$

$$e^{-y} = (1-e^{-1}) + \sum_{n=1}^{\infty}\left(\frac{2(1-e^{-1})}{4n^2\pi^2+1}\cos n\pi y + \frac{(1-e^{-1})4n\pi}{4n^2\pi^2+1}\sin 2n\pi y\right)\text{(III)}$$

$$e^{y-1} = (1-e^{-1}) + \sum_{n=1}^{\infty}\left(\frac{2(1-e^{-1})}{4n^2\pi^2+1}\cos 2n\pi y + \frac{(e^{-1}-1)4n\pi}{4n^2\pi^2}\sin 2n\pi y\right)\text{(IV)}$$

$$e^{1-y}=(e-1)+\sum_{n=1}^{\infty}\left(\frac{2(e-1)}{4n^2\pi^2+1}\cos 2n\pi y+\frac{(e-1)4n\pi}{4n^2\pi^2+1}\sin 2n\pi y\right)\quad (V)$$

$$e^{y+1}=(e^2-e)+\sum_{n=1}^{\infty}\frac{2(e^2-e)}{4n^2\pi^2+1}\cos 2n\pi y+\frac{(e-e^2)}{4n^2\pi^2+1}\sin 2n\pi y\quad (VI)$$

با جاگذاری عبارتهای (VI),(V),(IV),(VI),(II) در طرف راست عبارت (I) سری فوریه در تابع بدست می آید.

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48) $F(x,y)=(y^2-y)e^x \qquad ,0<x<2\pi \qquad ,0<y<2\pi$

$$F(x,y)=\frac{a_0(y)}{2}+\sum_{m=1}^{\infty}a_m(y)\cos\frac{m\pi}{a}x+b_m(y)\sin\frac{m\pi}{a}x \qquad ,a=\pi,b=\pi$$

$$F(x,y)=\frac{a_0(y)}{2}+\sum_{m=1}^{\infty}a_m(y)\cos mx+b_m(y)\sin mx$$

$$a_0(y)=\frac{1}{\pi}\int_0^{2\pi}(y^2-y)e^x dx=\frac{(y^2-y)}{\pi}(e^{2\pi}-1)$$

$$a_m(y)=\frac{1}{\pi}\int_0^{2\pi}(y^2-y)e^x\cos xdx\Rightarrow$$

$(+)e^x$	$\cos mx$
$(-)e^x$	$\frac{1}{m}\sin mx$
$(+)e^x$	$-\frac{1}{m^2}\cos mx$
	$\int -\frac{1}{m^2}\cos mx$

$$\int_0^{2\pi}e^x\cos mxdx=\left(\frac{e^x}{m}\sin mx+\frac{e^x}{m^2}\cos mx-\int_0^{2\pi}\frac{e^x}{m^2}\cos x\right)_0^{2\pi}$$

$$\Rightarrow\left(1+\frac{1}{m^2}\right)\int_0^{2\pi}e^x\cos 2m\pi xdx=\frac{(e^{2\pi}-1)}{m^2}\rightarrow a_m(y)=\frac{(y^2-y)}{\pi}\frac{(e^{2\pi}-1)}{(m^2+1)}$$

$$b_m(y)=\frac{1}{\pi}\int_0^{2\pi}(y^2-y)e^x\sin mxdx\Rightarrow$$

	$\sin mx$
$(+)e^{x}$	$\frac{-1}{m}\cos mx$
$(-)e^{x}$	$-\frac{1}{m^2}\sin mx$
$(+)e^{x}$	$\int\frac{-1}{m^2}\sin mx$

$$\int_0^{2\pi} e^{x}\sin mx=\left(\frac{-e^{x}}{m}\cos mx+\frac{e^{x}}{m^2}\sin mx+\int_0^{2\pi}\frac{-e^{x}}{m^2}\sin mx\right)_0^{2\pi}$$

$$\Rightarrow\int_0^{2\pi}\left(1+\frac{1}{m^2}\right)e^{x}\sin mx=\left(\frac{1-e^{2\pi}}{m}\right)\rightarrow b_m(y)=\frac{m(y^2-y)(1-e^{2\pi})}{\pi(m^2+1)}$$

$$F(x,y)=\frac{(y^2-y)(e^{2\pi}-1)}{2\pi}+\sum_{m=1}^{\infty}\frac{(y^2-y)(e^{2\pi}-1)}{\pi(m^2+1)}\cos mx+\frac{m(y^2-y)(1-e^{2\pi})}{\pi(m^2+1)}\sin mx \quad (\mathrm{I})$$

$$y^2=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos ny+b_n\sin ny)$$

$$a_0=\frac{1}{\pi}\int_0^{2\pi}y^2dy=\frac{8\pi^2}{3}$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}y^2\cos ny\,dy=\frac{1}{\pi}\left(\frac{y^2}{n}\sin ny+\frac{2y}{n^2}\cos ny-\frac{2}{n^3}\sin ny\right)_0^{2\pi}=\frac{4}{n^2}$$

$(+)y^2$	$\cos 2ny$
$(-)2y$	$\frac{1}{n}\sin ny$
$(+)2$	$-\frac{1}{n^2}\cos ny$
0	$-\frac{1}{n^3}\sin ny$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}y^2\sin ny\,dy=\frac{1}{\pi}\left(\frac{-y^2}{n}\cos ny+\frac{2y}{n^2}\sin ny+\frac{2}{n^3}\cos ny\right)_0^{2\pi}=\frac{-4\pi}{n}$$

$(+)y^2$	$\sin ny$
$(-)2y$	$-\frac{1}{2n}\cos ny$
$(+)2$	$-\frac{1}{n^2}\sin ny$
0	$\frac{1}{n^3}\cos ny$

$$y^2 = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos ny + \frac{-4\pi}{n}\sin ny\right) \text{(II)}$$

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos ny + b_n \sin ny)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} y\,dy = 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} y\cos ny\,dy = \frac{1}{\pi}\left(\frac{y}{n}\sin ny + \frac{1}{n}\cos ny\right)_0^{2\pi} = 0$$

$(+)y$	$\cos ny$
$(-)1$	$\frac{1}{n}\sin ny$
0	$-\frac{1}{n^2}\cos ny$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} y\sin ny\,dy = \frac{1}{\pi}\left(\frac{-y}{n}\cos ny + \frac{1}{n^2}\sin ny\right)_0^{2\pi} = \frac{-2}{n}$$

$(+)y$	$\sin ny$
$(-)1$	$-\frac{1}{n}\cos ny$
0	$-\frac{1}{n^2}\sin ny$

$$y = \pi + \sum_{n=1}^{\infty} \frac{-2}{n}\sin ny \quad \text{(III)}$$

با جاگذاری عبارات (II),(I) در عبارت (III) سری فوریه دوگانه تابع بدست می آید.

د) 49

$$F(x,y) = |x|\left(y^2 + \sin y\right) \qquad -1 < x < 1 \qquad ,0 < y < 2\pi$$

$$F(x,y) = \begin{cases} x\left(y^2 + \sin y\right) & 0 \le x < 1 \quad ,0 < y < 2\pi \\ -x\left(y^2 + \sin y\right) & -1 < x < 0 \quad ,0 < y < 2\pi \end{cases} \qquad \begin{matrix} a = 1 \\ b = \pi \end{matrix}$$

$$F(x,y) = \frac{a_0(y)}{2} + \sum_{n=1}^{\infty} a_n(y)\cos n\pi x + b_n(y)\sin n\pi x$$

$$a_n(y) = \int_{-1}^{0} -x(y^2 + \sin y)dx + \int_{0}^{1} x(y^2 + \sin y)dx = y^2 + \sin y$$

$$a_n(y) = \int_{-1}^{0} -x(y^2 + \sin y)\cos n\pi x dx + \int_{0}^{1} x(y^2 + \sin y)\cos n\pi x dx = \frac{2}{n^2\pi^2}(y^2 + \sin y)((-1)^n(-1)$$

$$b_n(y) = \int_{1}^{0} -x(y^2 + \sin y)\sin n\pi x dx + \int_{0}^{1} x(y^2 + \sin y)\sin bn\pi x dx = 0$$

$$F(x,y) = \frac{1}{2}(y^2 + \sin y) + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}(y^2 + \sin y)((-1)^n - 1)\cos n\pi x$$

$$y^2 + \sin y = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos my + b_m \sin my$$

$$a_0 = \frac{1}{\pi}\int_{0}^{2\pi}(y^2 + \sin y)dy = \frac{8\pi^2}{3}$$

$$a_m = \frac{1}{\pi}\int_{0}^{2\pi} y^2 \cos my dy + \frac{1}{\pi}\int_{0}^{2\pi} \sin y \cos my dy = \frac{4}{m^2}$$

$$b_m = \frac{1}{\pi}\int_{0}^{2\pi} y^2 \sin my dy + \frac{1}{\pi}\int_{0}^{2\pi} \sin y \sin my dy = \begin{cases} -4\pi + 1 & m = 1 \\ \frac{-4\pi}{m} & m \ne 1 \end{cases}$$

$$y^2+\sin y=\frac{4\pi^2}{3}+4\cos y+(-4\pi+1)\sin y+\sum_{\substack{m=1\\ m\neq 1}}^{\infty}\frac{4}{m^2}\cos my+\left(\frac{-4\pi}{m}\right)\sin my$$

$$y^2+\sin y=\frac{4\pi^2}{3}+\sin y+\sum_{m=1}^{\infty}\frac{4}{m^2}\cos my-\frac{4\pi}{m}\sin my$$

$$F(x,yt)=\tfrac{1}{2}\sin y+\frac{2\pi^2}{3}+\sum_{m=1}^{\infty}\frac{2}{m^2}\cos my-\frac{2\pi}{m}\sin my+2\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2\pi^2}\sin y\cos n\pi x+$$

$$\frac{8}{3}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cos n\pi x+8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^n-1}{n^2\pi^2}\cos n\pi x\left(\frac{1}{m^2}\cos my-\frac{\pi}{m}\sin my\right)$$

د (50)

$$F(x,y)=xy+1 \qquad -1<x<1 \qquad ,-l<y<l$$

$$a=1 \qquad b=2$$

$$F(x,y)=\frac{a,(y)}{2}+\sum_{n=1}^{\infty}a(y)\cos n\pi x+b(y)\sin n\pi x$$

$$a,(y)=\int_{-1}^{1}(xy+1)dx=2$$

$$a_n(y)=\int_{-1}^{1}(xy+1)\cos n\pi x dx=0$$

$$b_n(y)=\int_{-1}^{1}(xy+1)\sin n\pi x dx=\frac{2y}{n\pi}(-1)^n$$

$$F(x,y)=1+\sum_{n=1}^{\infty}\tfrac{2y}{n\pi}(-1)^n\sin n\pi x$$

د (51)

$$F(xy)=x^2y^2+yx^3 \qquad -\pi<x<\pi \qquad ,-1<y<1$$

$$a=\pi \qquad b=1$$

$$F(x,y) = \frac{a_0(y)}{2} + \sum_{n=1}^{\infty} a_n(y)\cos nx + b_n(y)\sin nx$$

$$a_0(y) = \frac{1}{\pi}\int_{-\pi}^{\pi}(x^2y^2 - yx^3)dx = \frac{2\pi^2}{3}y^2$$

$$a_n(y) = \frac{1}{\pi}\int_{-\pi}^{\pi}(x^2y^2 - yx^3)\cos nxdx = \frac{4y^2}{n^2}(-1)^n$$

$$b_n(y) = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(x^2y^2 - yx^3\right)\sin nxdx = 2y(-1)^n\left(\frac{\pi^2}{n} - \frac{6}{n^3}\right)$$

$$F(x,y) = \frac{\pi^2}{3}y^2 + \sum_{n=1}^{\infty}\frac{4y^2}{n^2}(-1)^n\cos nx + 2y(-1)^n\left(\frac{\pi^2}{n} - \frac{6}{n^3}\right)\sin nx$$

$$y^2 = \frac{1}{3} + \sum_{m=1}^{\infty}\frac{4}{m^2\pi^2}(-1)^m\cos m\pi y$$

$$y = \sum_{m=1}^{\infty}\left\{(-1)^{m+1}\frac{2}{m\pi} + ((-1)^m - 1)\frac{4}{m^3\pi^3}\right\}\sin m\pi y$$

$$F(x,y) = \frac{\pi^2}{9} + \frac{\pi^2}{3}\sum_{m=1}^{\infty}\frac{4}{m^2\pi^2}(-1)^m\cos m\pi y + \sum_{n=1}^{\infty}\frac{4(-1)^n}{3n^2}\cos nx +$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx\left\{\frac{4}{m^2\pi^2}(-1)^m\cos m\pi y\right\} +$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}2(-1)\left(\frac{\pi}{n} - \frac{6}{n}\right)\sin nx\left\{(-1)\frac{2}{m\pi} + ((-1)-1)\frac{4}{m\pi}\right\}\sin m\pi y$$

(52 (د

$$F(xy) = y^3x^2 + xy \qquad 0 < x < 2\pi \qquad 0 < y < 2$$