62)

$$F(x)=\begin{cases}0 & 1<|x| \\ 1+ax & -1<x<0 \\ 1+bx & 0<x<1\end{cases}$$

$$A(w)=\frac{1}{\pi}\int_{-\infty}^{\infty}F(x)\cos wx\,dx=\frac{1}{\pi}\left[\int_{-1}^{0}(1+ax)\cos wx\,dx+\int_{0}^{1}(1+bx)\cos wx\,dx\right]$$

$$=\frac{1}{\pi}\left[\left(\frac{1}{w}\sin wx+\frac{ax}{w}\sin wx+\frac{a}{w^2}\cos wx\right|_{-1}^{0}+$$

$$A(w)=\frac{1}{\pi}\left[\frac{a}{w^2}+\frac{1}{w}\sin w-\frac{a}{w}\sin w+\frac{1}{w}\sin w-\frac{a}{w^2}\cos w+\frac{b}{w}\sin w+\frac{b}{w^2}\cos w-\frac{b}{w^2}\right]\rightarrow$$

$$A(w)=\frac{1}{\pi}\left[\frac{b-a}{w^2}(\cos w-1)+\frac{b-a+2}{w}\sin w\right]$$

$$B(w)=\frac{1}{\pi}\int_{-\infty}^{\infty}F(x)\sin wx\,dx=\frac{1}{w}-\frac{1}{w}\cos w-\frac{a}{w}\cos w+\frac{a}{w^2}\sin w+$$

$$\frac{1}{w}\cos w-\frac{b}{w}\cos w+\frac{b}{w^2}\sin w-\frac{1}{w}\rightarrow$$

$$B(w)=\frac{1}{\pi}\left[\frac{a+b}{w}\cos w-\frac{\sin w}{w}\right]$$

$$F(x)=\int_{0}^{\infty}\left[(A(w))\cos wx+B(w)\sin w(x)\right]dw$$

63)

$$F(x)=\begin{cases} e^{a(x-b)} & -\infty < x < b, a>0 \\ 0 & b<x<\infty \end{cases}$$

$$A(w)=\frac{1}{\pi}\left(\int_{-\infty}^{b} e^{a(x-b)}\cos wx dx\right)=\frac{e^{-ab}}{\pi}\left(\int_{-\infty}^{b} e^{ax}\cos wx dx\right)=\frac{e^{-ab}}{\pi}\times I$$

$$I=\frac{e^{ax}}{w}\sin wx+\frac{ae^{ax}}{w^2}\cos wx-\frac{a^2}{w^2}I \rightarrow$$

$$A(w)=\frac{e^{-ab}}{\pi}\times I \rightarrow B(w)=\left(\frac{ae^{ab}}{w^2+a^2}\sin wb-\frac{w}{w^2+a^2}\cos wb\right)\frac{e^{-ab}}{\pi}$$

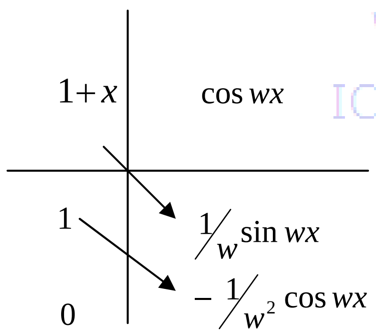
64)

$$F(x)=\begin{cases} x+1 & |x|<1 \\ 0 & |x|>1 \end{cases} \Rightarrow F(x)=\begin{cases} a+1 & -1<x<1 \\ 0 & \begin{cases} x>1 \\ x<-1 \end{cases} \end{cases}$$

$$F(x)=\int_{0}^{\infty}[A(w)\cos wx+B(w)\sin wx]dw$$

$$A(w)=\frac{1}{\pi}\int_{-\infty}^{\infty}F(x)\cos wx dx$$

$$A(w)=\frac{1}{\pi}\int_{-1}^{+1}(1+x)\cos wx dx=\frac{1}{\pi}\left[\frac{1+x}{w}\sin wx+\frac{1}{w^2}\cos wx\Big|_{-1}^{1}\right]$$



$$=\frac{1}{\pi}\left[\frac{2}{w}\sin w+\frac{1}{w^2}\cos w-\left[\frac{1}{w^2}\cos w\right]\right]$$

$$=\frac{1}{\pi}\left[\frac{2}{w}\sin w\right]$$

$$A(w)=\frac{1}{\pi}\left(\frac{2}{w}\sin w\right)$$

$$B(w)=\frac{1}{\pi}\int_{-\infty}^{\infty}F(x)\sin wx\,dx \Rightarrow \frac{1}{\pi}\int_{-1}^{1}(x+1)\sin wx\,dx = B(w)$$

$x+1$	$\sin wx$
1	$-\frac{1}{w}\cos wx$
0	$-\frac{1}{w^2}\sin wx$

$$B(w)=\frac{1}{\pi}\left[-\frac{(x+1)}{w}\cos wx+\frac{1}{w^2}\sin wx\Big|_{-1}^{1}\right]$$

$$=\frac{1}{\pi}\left[-\frac{2}{w}\cos w+\frac{1}{w^2}\sin w+\frac{1}{w^2}\sin w\right]$$

$$B(w)=\frac{1}{\pi}\left(-\frac{2}{w}\cos w+\frac{2}{w^2}\sin w\right)$$

$$F(x)=\frac{1}{\pi}\int_0^{\infty}\left[\left[\frac{2}{w}\sin w\right]\cos wx+\left[-\frac{2}{w}\cos w+\frac{2}{w^2}\sin w\right]\sin wx\right]dw$$

بررسی قضیه دریکله :

$x=1$ $\qquad \lim_{x\to 1^+}F(x)=0 \qquad \lim_{x\to 1^-}F(x)=2$

در $x=1$ ناپیوسته است پس باید سری فوریه به $F(x)$ میانگین حد چپ و راست تابع همگرا باشد.

$$\frac{2+0}{2}=1 \qquad \Rightarrow 1=\frac{1}{\pi}\int_0^{\infty}\left[\left[\frac{2}{w}\sin w\right]\cos w+\left[-\frac{2}{w}\cos w+\frac{2}{w^2}\sin w\right]\sin w\right]dw$$

$x=1$ $\qquad \lim_{x\to -1^+}F(x)=0 \qquad \lim_{x\to -1^-}F(x)=2$

در $x=1$ پیوسته است پس سری فوریه تناوب با تابع $F(x)$ به مقدار تابع همگرا باشد.

$$0=\frac{1}{\pi}\int_0^{\infty}\left[\left[\frac{2}{w}\sin w\right]\cos w-\left[-\frac{2}{w}\cos w+\frac{2}{w^2}\sin w\right]\sin w\right]dw$$

65)

$$F(x)=\begin{cases} a & -2<x<-1 \\ b & -1<x<1 \\ a & 1<x<2 \\ 0 & |x|>2 \end{cases}$$

$$A_1(w)=\int_{-2}^{-1} a\cos wx\,dx = a\left(\tfrac{1}{w}\sin wx\right|_{-2}^{-1} = \tfrac{a}{w}\left[-\sin w+\sin 2w\right]$$

$$A_2(w)=\int_{-1}^{1} b\cos wx\,dx = \left(\tfrac{1}{w}\sin wx\right|_{-1}^{1} = b\left[\tfrac{1}{w}\sin w+\tfrac{1}{w}\sin w\right]=\tfrac{2b}{w}\sin w$$

$$A_3(w)=a\int_{1}^{2}\cos wx\,dx=\tfrac{a}{w}\left[\sin wx\right|_{1}^{2}=\tfrac{a}{w}(\sin 2w-\sin w)\Rightarrow$$

$$A(w)=\tfrac{1}{\pi}\left(\tfrac{2a}{w}(\sin 2w-\sin w)+\tfrac{2b}{w}\sin w\right)$$

$$B_1(w)=\int_{-2}^{-1} a\sin wx\,dx = a\left[-\tfrac{1}{w}\cos wx\right|_{-2}^{-1}=-\tfrac{a}{w}\left[\cos w-\cos 2w\right]$$

$$B_2(w)=\int_{-1}^{1} b\sin wx\,dx=-\tfrac{b}{w}\left[\cos wx\right|_{-1}^{+1}=\cos w-\cos w=0$$

$$B_3(w)=\int_{1}^{2} a\sin wx\,dx=-\tfrac{a}{w}\left[\cos wx\right|_{1}^{2}=(\cos 2w-\cos w)-\tfrac{a}{w}$$

$$B_3(w)=\tfrac{a}{w}(\cos w-\cos 2w)$$

$$B(w)=\tfrac{a}{w}(\cos w-\cos 2w)-\tfrac{a}{w}(\cos w-\cos 2w)+0=0$$

$$F(x)=\tfrac{1}{\pi}\int_{0}^{\infty}\left[\tfrac{2}{w}a(\sin 2w-\sin w)+\tfrac{2b}{w}\sin w\right]\cos xdw$$

بررسی قضیه دیریکله :

$$\lim_{x \to -1^-} F(x) = a \qquad \lim_{x \to -1^+} F(x) = b$$

$a=-1 \leftarrow$ ناپیوسته : به میانگین حد چپ و راست همگرا است :

$$\frac{a+b}{2} = \frac{1}{\pi}\int_0^\infty \left[\frac{2}{w}a(\sin 2w - \sin w) + 2\frac{b}{w}[\sin w]\right]\cos dw$$

$x=3$ پیوسته است پس سری فوریه تناوب به مقدار تابع همگرا است.

$$\lim_{x \to 3^-} F(x) = 0 \qquad \lim_{x \to 3^+} F(x) = 0$$

$$0 = \frac{1}{\pi}\int_0^\infty \left[\frac{2}{w}a(\sin 2w - \sin w) + \frac{2b}{w}\sin w\right]dw$$

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66)

$$F(x) = \begin{cases} 1-x & -1 < x < 0 \\ x & 0 < x < 1 \\ 0 & \begin{cases} x > 1 \\ x < -1 \end{cases} \end{cases}$$

$$A_1(w) = \frac{1}{\pi}\int_{-1}^{0}(1-x)\cos wx\,dx = \frac{1}{\pi}\left[\frac{1-x}{w}\sin wx - \frac{1}{w^2}\cos wx\right]_{-1}^{0}$$

$1-x$	$\cos wx$
-1	$\frac{1}{w}\sin wx$
0	$-\frac{1}{w^2}\cos wx$

$$= \frac{1}{\pi}\left[-\frac{1}{w^2} - \left[-\frac{2}{w}\sin w - \frac{1}{w^2}\cos w\right]\right]$$

$$= \frac{1}{\pi}\left[-\frac{1}{w^2} + \frac{2}{w}\sin w + \frac{1}{w^2}\cos w\right]$$

$$A_2(w) = \frac{1}{\pi}\int_0^1 x\cos wx\,dx = \frac{1}{\pi}\left[\frac{x}{w}\sin wx + \frac{1}{w^2}\cos wx\right|_0^1$$

x	$\cos x$
1	$\frac{1}{w}\sin wx$
0	$-\frac{1}{w^2}\cos wx$

$$= \frac{1}{\pi}\left[\frac{1}{w}\sin w + \frac{1}{w^2}\cos w - \frac{1}{w^2}\right]$$

$$A(w) = \frac{1}{\pi}\left[\frac{3}{w}\sin w + \frac{2}{w^2}\cos w - \frac{2}{w^2}\right]$$

$$B_1(w) = \int_{-1}^{0}(1-x)\sin wx\,dx = -\frac{1-x}{w}\cos x - \frac{1}{w^2}\sin wx\Big|_{-1}^{0}$$

$1-x$	$\sin wx$
-1	$-\frac{1}{w}\cos x$
0	$-\frac{1}{w^2}\sin wx$

$$= +\frac{2}{w}\cos w - \frac{1}{w^2}\sin w + \left[-\frac{1}{w}\right]$$

$$B_1(w) = \frac{1}{\pi}\left[-\frac{1}{w} + \frac{2}{w}\cos w - \frac{1}{w^2}\sin w\right]$$

$$B_2(w)=\int_{-1}^{0}\sin wx dx=\left[-\frac{a}{w}\cos ex+\frac{1}{w^2}\sin wx\right|_0^1$$

x	$\sin wx$
1	$-\frac{1}{w}\cos wx$
0	$-\frac{1}{w^2}\sin wx$

$$=-\frac{1}{w}\cos w+\frac{1}{w^2}\sin w$$

$$B_2(w)=\frac{1}{\pi}\left[-\frac{1}{w}\cos w+\frac{1}{w^2}\sin w\right]$$

$$B(w)=\frac{1}{\pi}\left[\frac{1}{w}\cos w-\frac{1}{w}\right]$$

$$F(x)=\frac{1}{\pi}\int_0^\infty\left[\left[\frac{3}{w}\sin w+\frac{2}{w^2}\cos w-\frac{2}{w^2}\right]\cos wx+\left[\frac{1}{w}\cos w-\frac{1}{w}\right]\sin wx\right]dw$$

بررسی قضیه دیریکله:

$x=0$ $\qquad \lim_{x\to 0^+}F(x)=0 \qquad \lim_{x\to 0^-}F(x)=1$

ناپیوسته است پس سری فوریه به میانگین حد چپ و راست تابع همگراست.

$$\frac{1}{2}=\frac{1}{\pi}\int_0^\infty\left[\frac{3}{w}\sin w+\frac{2}{w^2}\cos w-\frac{2}{w^2}\right]dw$$

$x=2$ $\qquad \lim_{x\to 2^+}F(x)=0 \qquad \lim_{x\to 2^-}F(x)=0$

پیوسته و به مقدار تابع همگراست.

$$0=\frac{1}{\pi}\int_0^\infty\left[\left[\frac{3}{w}\sin w+\frac{2}{w^2}\cos w-\frac{2}{w^2}\right]\cos 2w+\left[\frac{1}{w}\cos w-\frac{1}{w}\right]\sin 2w\right]dw$$

67) $F(x)=\begin{cases} x-x^2 & 0\le x<1 \\ 0 & x\ge 1 \end{cases}$

نمایش انتگرال کسینوسی:

$$F(x)=\frac{2}{\pi}\int_0^\infty\int_0^\infty F(t)\cos wt\cos wx\,dt\,dw$$

$$F(x)=\frac{2}{\pi}\int_0^\infty\int_0^1 (t-t^2)\cos wt\cos wx\,t\,dw$$

$$\cos wx\int_0^1 (t-t^2)\cos wt\,dt$$

$$\cos x\int_0^1 (t-t^2)\cos wt\,dt=\cos wx\left[\frac{t-t^2}{w}\sin wt+\frac{1-2t}{w^2}\cos wt+\frac{2}{w^3}\sin wt\right|_0^1$$

$t-t^2$	$\cos wt$
$1-2t$	$\frac{1}{w}\sin wt$
-2	$-\frac{1}{w^2}\cos wt$
0	$-\frac{1}{w^3}\sin wt$

$$=\cos wx\left[-\frac{1}{w^2}\cos w+\frac{2}{w^3}\sin w-\frac{1}{w^2}\right]$$

نمایش انتگرالی cos ای فوریه تابع $F(x)$

$$F(x)=\frac{2}{\pi}\int_0^\infty\left[-\frac{\cos wx}{w^2}\cos w+\frac{2}{w^3}\sin w\cos wx-\frac{1}{w^2}\cos wx\right]dw$$

$$F(x)=\frac{2}{\pi}\int_0^\infty\int_0^\infty F(t)\sin wt\sin wx\,dt\,dw$$

$$\sin wx\int_0^1 (t-t^2)\sin wt\,dt$$

$$= \sin wx \int_0^1 (t - t^2) \sin wt\,dt = \sin wx \left[-\frac{t-t^2}{w} \cos wt + \frac{1-2t}{w^2} \sin wt - \frac{2}{w^3} \cos wt \right|_0^1$$

$$= \sin wx \left[-\tfrac{1}{w^2} \sin w - \tfrac{2}{w^3} \cos w + \tfrac{2}{w^3} \right]$$

$t - t^2$	$\sin wt$
$1 - 2t$	$-\frac{1}{w} \cos wt$
-2	$-\frac{1}{w^2} \sin wt$
0	$-\frac{1}{w^3} \cos wt$

نمایش انتگرالی sin ای فوریه تابع $F(x)$

$$F(x) = \tfrac{2}{\pi} \int_0^\infty \left[-\frac{\sin wx \sin w}{w^2} - \frac{2 \sin wx \cos w}{w^3} + \tfrac{2}{w^3} \sin wx \right] dw$$

بررسی قضیه دیریکله :

ی ا cos $\rightarrow$ $\lim_{x \to 1^+} F(x) =$ $\lim_{x \to 1^-} F(x) =$

پیوسته است و تعداد تابع همگراست.

$x = 1$

$$0 = \tfrac{2}{\pi} \int_0^\infty \left[-\frac{\cos w}{w^2} \cos w + \frac{2}{w^3} \sin w \cos w - \frac{1}{w^2} \cos w \right] dw$$

$0 = 0$ پیوسته $x = 0$

ح) 68)

$$F(x)=\begin{cases}1+\cos x & 0\le x\le \pi \\ 0 & \pi \le x<\infty\end{cases} \qquad 2l=\pi$$

$$F(x)=\int_0^\infty [A(w)\cos wx + B(w)\sin wx]dw$$

$$A(w)=\cos x$$

$$B(w)=\frac{2}{\pi}\left[\frac{1-\cos w\pi}{w}-\frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1}+\frac{\cos(w-1)\pi}{w-1}+\frac{w}{w^2-1}\right)\right]$$

$$0=\int_0^\infty\left\{\cos x.\cos wx+\frac{2}{\pi}\left[\frac{1-\cos w\pi}{w}-\frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1}+\frac{\cos(w-1)\pi}{w-1}+\frac{w}{w^2-1}\right)\right]\sin wn\right\}dw$$

در $x=\pi$ پیوسته . انتگرال فوریه برابر مقدار تابع است.

د $$0=\int_0^\infty\left\{-\cos w\pi+\frac{2}{\pi}\left[\frac{1-\cos w\pi}{w}-\frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1}+\frac{\cos(w-1)\pi}{w-1}+\frac{w}{w^2-1}\right)\right]\sin w\pi\right\}dw$$

ر $x=\frac{\pi}{2}$ پیوسته .

$$1=\int_0^\infty\frac{2}{\pi}\left[\frac{1-\cos w\pi}{w}-\frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1}+\frac{\cos(w-1)\pi}{w-1}+\frac{w}{w^2-1}\right)\right]\sin w\frac{\pi}{2}dw$$

$$\frac{2}{\pi}=\int_0^\infty\left[\frac{1-\cos w\pi}{w}-\frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1}+\frac{\cos(w-1)\pi}{w-1}+\frac{w}{w^2-1}\right)\right]\sin w\frac{\pi}{2}dw$$

د) 68)

$$F(x)=\begin{cases}1+\cos x & 0\le x\le \pi \\ 0 & \pi \le x>\infty\end{cases}$$

$$F(x)=\int_0^\infty A(w)\cos wxdw$$

$$A(w)=\frac{2}{\pi}\int_0^\pi F(x)\cos wxdx=\frac{2}{\pi}\int_0^\pi (1+\cos x)\cos wxdx$$

$$=\frac{2}{\pi}\left[\int_0^\pi \cos wxdx+\int_0^\pi \cos x.\cos wxdx\right]$$