$$= \frac{2}{\pi}\left(-\frac{1}{w}\sin wn \Big|_0^{\pi} + \begin{cases} 0 & w \neq 1 \\ \frac{\pi}{2} & w = 1 \end{cases} \right. \qquad = \begin{cases} 0 & w \neq 1 \\ 1 & w = 1 \end{cases}$$

$$F(x) = \int_0^{\infty} \cos xdw = \cos nw \Big|_{w=1} = \cos x \rightarrow$$ نمایش انتگرال کسینوسی

$$F(x) = \int_0^{\infty} B(w) \sin wxdw$$

$$B(w) = \frac{2}{\pi}\int_0^{\pi} F(x)\sin wxdx = \frac{2}{\pi}\int_0^{\pi}(1+\cos x)\sin wxdx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi}\sin wxdx + \int_0^{\pi}\cos x \sin wxdx\right]$$

$$= \frac{2}{\pi}\left[\frac{-1}{w}\cos wx\Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi}[\sin(w+1)n + \sin(w-1)n]dx\right]$$

$$= \frac{2}{\pi}\left\{\frac{-1}{w}\cos w\pi + \frac{1}{w} + \frac{1}{2}\left[\frac{-\cos(w+1)x}{w+1} + \frac{-\cos(w-1)x}{w-1}\Big|_0^{\pi}\right\}\right.$$

$$= \frac{2}{\pi}\left[\frac{1}{w} - \frac{1}{w}\cos w\pi - \frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1} + \frac{\cos(w-1)\pi}{w-1} - \frac{2w}{w^2-1}\right)\right]$$

$$F(x) = \frac{2}{\pi}\int_0^{\infty}\left[\frac{1-\cos w\pi}{w} - \frac{1}{2}\left(\frac{\cos(w+1)\pi}{w+1} + \frac{\cos(w-1)\pi}{w-1}\right) + \frac{w}{w^2-1}\right]\sin wxdw$$

↑ نمایش سینوسی

ح) 69)

$$F(x)=\begin{cases}1+\sin x & 0<x<\pi\\ 2-\dfrac{x}{\pi} & \pi<x\le\pi\end{cases}$$

نمایش انتگرال استاندارد

$$F(x)=\int_0^\infty [A(w)\cos wx+B(w)\sin wx]dw$$

$$F(x)=\frac{2}{\pi}\int_0^\infty\left\{\left[\frac{-\cos\pi w-1}{1-w^2}-\frac{1}{\pi w^2}(\cos 2\pi w-\cos\pi w\right]\cos wx+\frac{1}{w\sin wx}\right\}dw$$

در $n=\pi$ ناپیوسته است پس باید انتگرال فوریه به میانگین حد چپ و راست میل کند.

$$\frac{1+1}{2}=\frac{2}{\pi}\int_0^\infty\left\{\left[\frac{-\cos\pi w}{1-w^2}-\frac{1}{\pi w^2}(\cos 2\pi w-\cos\pi w)\right]\cos w\pi+\frac{1}{w}\sin w\pi\right\}dw$$

$$\frac{\pi}{2}=\int_0^\infty\left\{\left[\frac{-\cos\pi w-1}{1-w^2}-\frac{1}{\pi w^2}(\cos 2\pi w-\cos\pi w)\right]\cos w\pi+\frac{\sin w\pi}{w}\right\}dw$$

در $n=\frac{\pi}{2}$ پیوسته

$$\pi=\int_0^\infty\left\{\left[\frac{-\cos\pi w-1}{1-w^2}-\frac{1}{\pi w^2}(\cos 2\pi w-\cos\pi w)\right]\cos\frac{w\pi}{2}+\frac{\sin w\pi}{2w}\right\}dw$$

ز) 69

$$F(x)=\begin{cases}1+\sin x & 0<x<\pi\\ 2-\dfrac{x}{\pi} & \pi<x\le 2\pi\end{cases}$$

$$F(x)=\int_0^\infty A(w)\cos wx\,dw \qquad A(w)=\frac{2}{\pi}\int_0^\infty F(x)\cos wx\,dx$$

$$A(w)=\frac{2}{\pi}\left[\int_0^\pi(1+\sin x)\cos wx\,dx+\int_\pi^{2\pi}\left(2-\frac{x}{\pi}\right)\cos wx\,dx\right]$$

$$= \frac{2}{\pi}\left\{ \frac{-1}{w}\sin wn\Big|_0^{\pi} + \int_0^{\pi}\sin x\cos wx\,dx + \frac{2}{w}\sin wx\Big|_{\pi}^{2\pi} - \frac{1}{\pi}\int_{\pi}^{2\pi} x\cos wx\,dx \right\}$$

$$= \frac{2}{\pi}\left[\frac{-\cos w\pi - 1}{1-w^2} - \frac{1}{\pi w^2}(\cos 2\pi w - \cos \pi w) \right]$$

$$\int_0^{\pi}\sin x\cos wx\,dx = \frac{1}{2}\int_0^{\pi}\left[\sin(1+w)x + \sin(1-w)x\right]dx$$

$$= \frac{-1}{2}\left[\frac{\cos(1+w)x}{1+w} + \frac{\cos(1-w)x}{1-w}\right]_0^{\pi} = -\frac{1}{2}\left[\frac{\cos(1+w)\pi}{1+w} + \frac{\cos(1-w)\pi}{1-w} - \left(\frac{2}{1-w^2}\right)\right]$$

$$= -\frac{1}{2}\left[\frac{\cos\pi,\cos w\pi - \sin\pi,\sin w\pi}{1+w} + \frac{-\sin\pi.\sin w\pi + \cos\pi\cos w\pi}{1-w} - \left(\frac{2}{1-w^2}\right)\right.$$

$$= \frac{1}{2}\left[\frac{-\cos\pi}{1+w} - \frac{\cos w\pi}{1-w} - \frac{2}{1-w^2}\right] = \frac{1}{2}\left[\frac{-\cos w\pi + w\cos w\pi + \cos w\pi + w\cos w\pi}{1-w^2} - \frac{2}{1-w^2}\right]$$

$$= \frac{1}{2}\times 2\left[\frac{-\cos w\pi - 1}{1-w^2}\right] = \frac{-\cos w\pi - 1}{1-w^2}$$

P	I
x	$\cos wx$
1	$\frac{1}{w}\sin wx$
0	$-\frac{1}{w^2}\cos wx$

$$\int_{\pi}^{2\pi} x\cos wx\,dx = \left[\frac{x}{w}\sin wx + \frac{1}{w^2}\cos wx\right]_{\pi}^{2\pi}$$

$$= \frac{1}{w^2}\left[\cos w2\pi - \cos w\pi\right]$$

$$F(x) = \int_0^{\infty}\frac{2}{\pi}\left[\frac{-\cos \pi w - 1}{1-w^2} - \frac{1}{\pi w^2}(\cos 2\pi w - \cos \pi w)\right].\cos wx\,dw$$

↑

نمایش کوسینوسی

$$F(x)=\int_0^\infty B(w)\sin wx\,dw \qquad B(w)=\frac{2}{\pi}\int_0^\infty F(x)\sin wx\,dx$$

$$B(w)=\frac{2}{\pi}\int_0^\pi (1+\sin n)\sin wx\,dx+\int_\pi^{2\pi}\left(2-\frac{x}{\pi}\right)\sin wx\,dx$$

$$=\frac{2}{\pi}\left[-\frac{1}{w}\cos wx\Big|_0^\pi+\int_0^\pi \sin x.\sin wx\,dx-\frac{2}{w}\cos wx\Big|_\pi^{2\pi}-\frac{1}{\pi}\int_\pi^{2\pi} x\sin wx\,dx\right]$$

$$=\frac{2}{\pi}\left[\frac{-1}{w}(\cos\pi w-1)-\frac{2}{w}(\cos 2\pi w-\cos\pi w)+\frac{1}{w}(2\cos 2\pi w-\cos\pi w)\right.$$

$$=\frac{2}{\pi w}$$

$$\int_0^\pi \sin x.\sin wx\,dx=\frac{1}{2}\int_0^\pi[\cos(1+w)n-\cos(1-w)x]dx$$

$$=\frac{-1}{2}\left[\frac{\sin(1+w)x}{1+w}+\frac{\sin(1-w)x}{1-w}\right]_0^\pi=0$$

$$\int_\pi^{2\pi} x.\sin wx\,dx=\left[-\frac{x}{w}\cos wx+\frac{1}{w^2}\sin wn\right]_\pi^{2\pi}$$

$$=-\frac{2\pi}{w}\cos 2\pi w+\frac{\pi}{w}\cos\pi w=-\frac{\pi}{w}(2\cos 2\pi w-\cos\pi w)$$

P	I
$+x$	$\sin wx$
-1	$-\frac{1}{w}\cos wn$
0	$+\frac{1}{w^2}\sin wx$

$$F(x)=\int_0^\infty \frac{2}{\pi w}\sin wx\,dw$$

نمایش سینوسی

70)

$$F(x)=\begin{cases}\dfrac{2-x}{e} & 0\le x<1\\ e^{-x} & 1<x<\infty\end{cases}$$

$$F(x)=\int_0^\infty [A(w)\cos wn + B(w)\sin wx]dw$$

$$F(x)=\frac{2}{e\pi}\int_0^\infty \left\{\left[\frac{w\sin w-\cos w}{w^2}-\frac{w\sin w+\cos w}{w^2+1}\right]\cos wx + \left[\frac{-w.\cos-\sin w-2w}{w^2}+\frac{w\cos w+\sin w}{1+w^2}\right]\sin wx\right\}dw$$

در $n=1$ ناپیوسته پس انتگرال فوریه به میانگین حد چپ و راست میل می کند.

$$\frac{\frac{1}{e}+e^{-1}}{2}=\frac{2}{e\pi}\int_0^\infty \left\{\left[\frac{w\sin w-\cos w}{w^2}-\frac{w\sin w+\cos w}{w^2+1}\right]\cos w + \left[\frac{-w\cos w-\sin-2w}{w^2}+\frac{w\cos w+\sin w}{1+w^2}\right]\sin w\right\}dw$$

$$\frac{\pi}{2}=\int_0^\infty \left\{\left[\frac{w\sin w-\cos w}{w^2}-\frac{w\sin w+\cos w}{w^2+1}\right]\cos w + \left[\frac{-w\cos w-\sin w-2w}{w^2}+\frac{w\cos w+\sin w}{1+w^2}\right]\sin w\right\}dw$$

در $n=2$ پیوسته . پس انتگرال فوریه به مقدار تابع میل می کند.

$$\frac{\pi}{2e}=\int_0^\infty \left\{\left[\frac{w\sin w-\cos w}{w^2}-\frac{w\sin w+\sin w}{1+w^2}\right]\cos 2w + \left[\frac{-w\cos w-\sin w-2w}{w^2}+\frac{w\cos w+\sin w}{1+w^2}\right]\sin 2w\right\}dw$$

ز (70)

$$F(x)=\begin{cases}\dfrac{2-n}{e} & 0\le x\le 1\\ e^{-x} & 1<x<\infty\end{cases}$$

$$F(x)=\int_0^\infty A(w)\cos wx\,dw$$

$$A(w)=\frac{2}{\pi}\left[\int_0^1\left(\frac{2-x}{e}\right)\cos wx\,dx+\int_1^\infty e^{-x}.\cos wx\,dx\right]$$

I

$$=\frac{2}{\pi e}\left[\int_0^1 2\cos wx\,dx-\int_0^1 x\cos wx\,dx\right]+\frac{2}{\pi}\int_1^\infty e^{-x}\cos wx\,dx$$

P	I
x	$\cos wx$
1	$\frac{1}{w}\sin wx$
0	$-\frac{1}{w^2}\cos wx$

$$=\frac{2}{\pi e}\left[\frac{2}{w}\sin wx-\left(\frac{x}{w}\sin wx+\frac{x}{w^2}\cos wx\right)\right]_0^1$$

$$-\frac{2}{\pi}.\frac{w}{w^2+1}\left(\frac{\sin w}{e}-\frac{\cos w}{we}\right)=\frac{2}{e\pi}\left(\frac{1}{w}\right)\left(\sin w-\frac{\cos w}{w}\right)$$

$$-\frac{2}{e\pi}\left(\frac{w}{w^2+1}\right)\left(\sin w-\frac{\cos w}{w}\right)=\frac{2}{e\pi}\left(\sin w-\frac{\cos w}{w}\right)\left(\frac{1}{w}-\frac{w}{w^2+1}\right)$$

P I

$$I=\int_1^\infty e^{-x}.\cos wxdx=\frac{1}{w}e^{-x}\sin wx-\frac{1}{w^2}e^{-x}\cos wx\Big|_1^\infty$$

$$-\frac{1}{w^2}\int_1^\infty e^{-x}\cos wxdx$$

$$\left(1+\frac{1}{w^2}\right)I=\left[\frac{1}{w}e^{-x}\sin wn-\frac{1}{w}e^{-x}\cos wn\right]$$

$$I=\frac{w}{w^2+1}\left(e^{-x}\sin wn-\frac{1}{w}e^{-x}\cos wx\Big|_1^\infty=-\frac{w}{w^2+1}\left(\frac{\sin w}{e}-\frac{\cos w}{we}\right)\right.$$

	$\cos wx$
e^{-x}	$\frac{1}{w}\sin wx$
$-e^{-x}$	$-\frac{1}{w^2}\cos wn$
e^{-x}	$-\frac{1}{w^2}\int\cos wx$

نمایش انتگرال کوسینوسی

$$F(x)=\frac{2}{e\pi}\left[\int_0^\infty\frac{\sin w}{w}.\cos wxdw-\int_0^\infty\frac{\cos w}{w^2}.\cos wxdw\right.$$

$$\left.-\int_0^\infty\frac{w\sin w}{w^2+1}.\cos wxdw-\int_0^\infty\frac{\cos w}{w^2+1}\cos wxdw\right]$$

$$=\frac{2}{e\pi}\left[\int_0^\infty\frac{w\sin w-\cos w}{w^2}.\cos wxdw-\int_0^\infty\frac{w\sin w+\cos w}{w^2+1}.\cos wxdw\right]$$

$$F(x)=\int_0^\infty B(w)\sin wxdw$$

$$B(w)=\frac{2}{\pi}\int_0^1\frac{2-x}{e}\sin wxdx+\int_1^\infty e^{-x}\sin wxdx$$

$$=\frac{2}{\pi e}\left[\int_0^1 2\sin wxdx-\int_0^1 x\sin wxdx+\frac{2}{\pi}\int_1^\infty e^{-x}\sin wxdx\right.$$

P	I
x	$\sin wx$
1	$-\frac{1}{w}\cos wx$
0	$-\frac{1}{w^2}\sin wx$

$$=\frac{2}{\pi e}\left[-\frac{2}{w}\cos wx+\frac{x}{w}\cos wx-\frac{1}{w^2}\sin wx\Big|_0^1\right.$$

$$+\frac{2}{\pi e}\left(\frac{w}{1+w^2}\right)\left(\cos w+\frac{1}{w}\sin w\right)=\frac{2}{\pi e}\left[\frac{\cos w}{w}-\frac{\sin w}{w^2}-\frac{2}{w}\right]$$

$$+\frac{2}{\pi e}\left(\frac{w}{1+w^2}\right)\left(\cos w+\frac{1}{w}\sin w\right)$$

P	I
e^{-x}	$\sin wx$
$-e^{-x}$	$-\frac{1}{w}\cos wx$
e^{-x}	$-\frac{1}{w^2}\sin wx$
	$-\frac{1}{w^2}\int \sin wx$

$$I=\int_1^{\infty} e^{-x}\sin wx dx=\left[-\frac{e^{-x}}{w}\cos wx-\frac{e^{-x}}{w}\sin wx\right]_1^{\infty}-\frac{1}{w^2}\int_1^{\infty} e^{-x}\sin wx$$

$$\left(1+\frac{1}{w^2}\right)I=-\left(-\frac{1}{ew}\cos w-\frac{1}{ew^2}\sin w\right)$$

$$I=\frac{w}{1+w^2}\left(\frac{1}{e}\cos w+\frac{1}{ew}\sin w\right)$$

نمایش انتگرال سینوسی

$$F(x)=\frac{2}{\pi e}\left[\int_0^{\infty}\frac{-w.\cos w-\sin w-2w}{w^2}.\sin wxdw+\int_0^{\infty}\frac{w.\cos w+\sin w}{1+w^2}.\sin wxdw\right]$$

(71 (ز

$$F(x)=\begin{cases} xe^{-x} & 0<x<\pi \\ 0 & \pi<x<\infty \end{cases}$$

$$F(x)=\int_0^\infty A(w)\cos wxdw$$

$$A(w)=\frac{2}{\pi}\int_0^\pi xe^{-x}.\cos wxdx$$

$$l=(x.\cos wx)$$

$$=\frac{2}{\pi}\int_0^\pi e^{-x}(x.\cos wx)dx=\frac{2}{\pi}l(x.\cos wx)=\frac{2d}{\pi dp}\left(\frac{p}{p^2+w^2}\right)=\frac{2}{\pi}\left(\frac{w^2-p^2}{(p^2+w^2)^2}\right)$$

$$F(x)=\frac{2}{\pi}\int_0^\infty \frac{(w^2-p^2)}{(p^2+w^2)^2}.\cos wx.dw=\frac{2}{\pi}\int_0^\infty \frac{(w^2-p^2)}{(p^2+w^2)^2}.\frac{e^{wx}-e^{-wx}}{2}.dw$$

نمایش انتگرال سینوسی

$$\frac{1}{\pi}\int_0^\infty \frac{(w^2-p^2)}{(p^2+w^2)^2}e^{wx}dw+\frac{1}{\pi}\int_0^\infty \frac{(w^2-p^2)}{(p^2+w^2)^2}e^{-wx}dw$$

$$l(x.\sin wx)$$

$$B(w)=\frac{2}{\pi}\int_0^\pi xe^{-x}\sin wxdx=\frac{2}{\pi}\int_0^\pi e^{-x}(e\sin x)dx=\frac{2}{\pi}.\frac{d}{dp}\left(\frac{1}{p^2+w^2}\right)$$

$$=\frac{2}{\pi}\left(\frac{-2p}{(p^2+w^2)^2}\right)=\frac{-4}{\pi}\frac{p}{(p^2+w^2)^2}$$

$$F(x) = -\frac{4}{\pi}\int_0^\infty \frac{p}{(p^2+w^2)^2}.\sin wx.dw = \frac{-4}{\pi}\int_0^\infty \frac{p}{(p^2+w^2)^2}.\frac{e^{wx}+e^{-wx}}{2i}dw$$

نمایش انتگرال سینوسی

ح) 71)

$$F(x) = \begin{cases} xe^{-x} & 0 < x < \pi \\ 0 & \pi < x < \infty \end{cases}$$

$$F(x) = \int_0^\infty (A(w)\cos wx + B(w)\sin wx)dw$$

$$F(x) = \int_0^\infty \left(\frac{2}{\pi}\frac{w^2-p^2}{(w^2-p^2)}.\cos wx - \frac{4}{\pi}\frac{p}{(w^2+p^2)}\sin wx\right)dw$$

در $x=\pi$ ناپیوسته است پس انتگرال فوریه میانگین حد چپ و راست میل می کند.

$$\frac{\pi e^{-\pi}+0}{2} = \frac{2}{\pi}\int_0^\infty \left(\frac{w^2-p^2}{(w^2+p^2)^2}.\cos w\pi - \frac{2p}{(w^2+p^2)^2}\sin w\pi\right)dw$$

$$\frac{\pi^2}{4e^\pi} = \int_0^\infty \left(\frac{w^2-p^2}{(w^2-p^2)^2}.\cos w\pi - \frac{2p}{(w^2+p^2)^2}\sin w\pi\right)dw$$

در $x=2\pi$ پیوسته است پس انتگرال فوریه به مقدار تابع میل می کند.