$$0=\int_0^\infty\left(\frac{w^2-p^2}{(w^2-p^2)^2}\cos 2\pi w-\frac{2p}{(w^2+p^2)^2}\sin 2\pi w\right)dw$$

---

78)

$$F(w)=\frac{1-e^{-3iw}}{2+iw}$$

$$F^{-1}\left(\frac{1}{2+iw}\right)=e^{-2x}+H(x),F^{-1}\left(\frac{-e^{-3iw}}{2+iw}\right)=-e^{-2(x-3)}H(x-3)$$

$$F(x)=\frac{1}{2+iw}-\frac{e^{-3iw}}{2-iw}\rightarrow F(x)=e^{-2x}H(x)-e^{-2(x-3)}H(x-3)$$

---

79)

$$F(w)=\frac{e^{-iw}}{2\pi(1+iw)(2+iw)}$$

$$F(w)=\frac{e^{-iw}}{2\pi}\left(\frac{1}{1+iw}-\frac{1}{2+iw}\right)=\frac{1}{2\pi}\left(\frac{e^{-iw}}{1+iw}-\frac{e^{-iw}}{2+iw}\right)$$

$$F(x)={}^{1}\!/_{2\pi}\left(e^{-(x-1)}H(x-1)-e^{-2(x-1)}H(x-1)\right)$$

جواب تمرین 75)

$$F(x)=\begin{cases}e^x & 0<x<1\\ 0 & 1<x<\infty\end{cases}$$

انتگرال کسینوسی: $B(w)=0$

$$A(w)=\frac{2}{\pi}\int_0^1 e^x\cos wx\,dx$$

$$=\frac{2}{\pi}\left[\frac{e^x}{w}\sin wx+\frac{e^x}{w^2}\cos wx\right]_0^1-\frac{2}{\pi w}\int_0^1 e^x\cos wx\,dx$$

$$\Rightarrow A(w)=\frac{2}{\pi}\left[\frac{e}{w}\sin w+\frac{e}{w}\cos w-\frac{1}{w}\right]-\frac{1}{w^2}A(w)$$

$$\Rightarrow\left(1+\frac{1}{w^2}\right)A(w)=\frac{2}{\pi}\left(\frac{ew\sin w+e\cos w-1}{w^2}\right)$$

$$\Rightarrow A(w)=\frac{2}{\pi}\left(\frac{ew\sin w+e\cos w-1}{w^2+1}\right)$$

$$F(x)=\int_0^\infty A(w)\cos wx\,dw$$

بررسی قضیه دیریکله :

$$x=0\Rightarrow F(0)=1=\int_0^\infty A(w)\cos(0)dw\Rightarrow\frac{\pi}{2}=\int_0^\infty\frac{ew\sin w+e\cos w-1}{w^2+1}dw$$

$$x=1\Rightarrow\frac{0+e}{2}=\int_0^\infty A(w)\cos w\,dw\Rightarrow\frac{\pi e}{4}=\int_0^\infty\frac{ew\sin w+e\cos w-1}{w^2+1}\cos w\,dw$$

انتگرال سینوسی: $A(w)=0$

$$B(w)=\frac{2}{\pi}\int_0^1 e\sin wx\,dx=\frac{2}{\pi}\left[-\frac{e^x}{w}\cos wx+\frac{e^x}{w^2}\sin wx\right]_0^1-\frac{1}{w^2}B(w)$$

$$B(w)=\frac{2}{\pi}\left(\frac{ew\cos w+e\sin w+w}{w^2+1}\right)$$

$$F(x)=\int_0^\infty B(w)\sin wx\,dw$$

بررسی قضیه دیریکله :

$$x=0 \to \frac{1-1}{2}=0=\int_0^\infty B(w)\sin(0)dw \Rightarrow 0=0$$

$$x=1 \to \frac{e+O}{2}=\frac{e}{2}=\int_0^\infty B(w)\sin wdw \Rightarrow \frac{e\pi}{4}=\int_0^\infty \frac{-ew\cos w+e\sin w+w}{w^2+1}\sin wdw$$

جواب تمرین 86

$$F(w)=\frac{4a\sin kw}{w(a^2+w^2)} \to F(x)=?$$

$$F(w)=\frac{4a\sin kw}{w(a^2+w^2)}=\frac{4ae^{ike}}{2iw(a^2+w^2)}-\frac{4ae^{-ike}}{2iw(a^2+w^2)}$$

$$\Rightarrow F(w)=\frac{2iae^{-ike}}{w(a^2+w^2)}-\frac{2iae^{ike}}{w(a^2+w^2)}$$

$$F^{-1}\left(\frac{2a}{a^2+w^2}\right)=e^{-a|x|}$$

$$F^{-1}\left(\frac{F(w)}{w}\right)=? \qquad \frac{F(w)}{w}=G(w) \to F(w)=waG(w)$$

وارون $\downarrow$ $\quad if(w)=iwG(w)$

$\downarrow$ $\quad if\ /(x)=g'(x)$

$$g(x)=F^{-1}(G(w))=i\int f(x)dx+C$$

پس $F^{-1}\left(\frac{F(w)}{w}\right)=i\int F(x)dx+C$

بنابراین:

$$F^{-1}\left(\frac{2a}{w(a^2+w^2)}\right)=i\int e^{-a|x|}dx+C_1 \qquad \int e^{-a|x|}=\begin{cases}\int e^{-ax}dx=\frac{-1}{a}e^{-ax} & x>0\\ \int e^{ax}dx=\frac{1}{a^{ax}} & x<0\end{cases}=\frac{-1}{a}e^{-a|x|}\frac{|x|}{x}$$

$$F^{-1}\left(\frac{2a}{w(w^2+a^2)}\right)=i\left(\frac{-1}{a}e^{-a|x|}\frac{|x|}{x}+C_1\right)$$

$$F^{-1}\left(\frac{2ia}{w(w^2+a^2)}\right)=i^2\left(\frac{-1}{a}e^{-a|x|}\frac{|x|}{x}+C_1\right)=\left(\frac{1}{a}e^{-a|x|}\frac{|x|}{x}-C_1\right)$$

$$F^{-1}\left(\frac{2iae^{-ikw}}{w(w^2+a^2)}\right)=\left(\frac{1}{a}e^{-a|x-k|}\frac{|x-k|}{x-k}-C_1\right)$$

همچنین: $F^{-1}\left(\frac{2iae^{ikw}}{w(w^2+a^2)}\right)=\left(\frac{1}{a}e^{-a|x+k|}\frac{|x+k|}{x+k}-C_1\right)$

پس: $F(w)=\frac{1}{a}\left(e^{-a|x-k|}\frac{|x-k|}{x-k}-e^{-a|x+k|}\frac{|x+k|}{x+k}\right)$

جواب تمرین 84)

$$F(w)=\frac{e^{-iw}}{1+w^2}\rightarrow F(x)=?$$

$$F^{-1}\left(\frac{2a}{a^2+w^2}\right)=e^{-a|x|}$$

$a=1$ $\longrightarrow$ $F^{-1}\left(\frac{2}{1+w2}\right)=e^{-|x|}$

$$\Rightarrow F^{-1}\left(\frac{1}{1+w^2}\right)=\frac{1}{2}e^{-|x|}$$

$x\rightarrow x-1$ $\longrightarrow$ $F^{-1}\left(\frac{e^{-iw}}{1+w^2}\right)=\frac{1}{2}e^{-|x-1|}$

$$\Rightarrow F(x)=\frac{1}{2}e^{-|x-1|}$$

---

76)

$$F(w)=\frac{e^{-2iw}}{3(1+iw)} \to F^{-1}\{F(w)\}=\tfrac{1}{3}e^{-(x-2)} \qquad H(x-2)$$

---

77)

$$F(w)=\frac{e^{3iw}}{\pi(2+iw)}=\frac{1}{\pi}e^{-2(x+3)} \qquad H(x+3)$$

جواب تمرین 85)

$$F(x)=\begin{cases}\sin x & 0\le x\le \pi \\ 0 & elsewhere\end{cases} \quad \to F(w)=?$$

$$F(w)=\int_{-\infty}^{\infty}F(x)e^{-iwx}dx=\int_{0}^{\pi}\sin x e^{-iwx}dx$$

$$=\int_0^{\pi}\frac{e^{ix}-e^{-ix}}{2i}e^{-iwx}dx$$

$$=\int_0^{\pi}\frac{e^{-x}\times e^{-iwx}}{2i}dx-\int_0^{\pi}\frac{e^{-ix}\times e^{-iwx}}{2i}dx$$

$$=\frac{1}{2i}\int_0^{\pi}e^{ix(1-w)}dx-\frac{1}{2i}\int_0^{\pi}e^{-ix(1+w)}dx$$

$$=\frac{1}{2i}\frac{e^{ix(1-w)}}{i(1-w)}\Bigg|_0^{\pi}-\frac{1}{2i}\frac{e^{-ix(1+w)}}{-i(1+e)}\Bigg|_0^{\pi}$$

$$=\frac{1}{2(w-1)}(e^{i\pi(1-w)}-1)-\frac{1}{2(1+w)}(e^{-i\pi(1+w)}-1)$$

$$e^{i\pi(1-w)}=\cos\pi(1-w)+i\sin\pi(1-w)=\cos(\pi-\pi w)+i\sin(\pi-\pi w)=-\cos\pi w+i\sin\pi w$$

$$e^{i\pi(1+w)}=\cos\pi(1-w)-i\sin\pi(1+w)=\cos(\pi+\pi w)-i\sin(\pi+\pi w)=-\cos\pi w+i\sin\pi w$$

پس با هم برابرند

بنابراین:

$$F(w)=\left(\frac{e^{i\pi(1-w)}-1}{2}\right)\left(\frac{1}{w-1}-\frac{1}{w+1}\right)=\frac{e^{i\pi(1-w)}-1}{2}\times\frac{2}{w^2-1}$$

$$F(w)=\frac{e^{i\pi(1-w)}-1}{w-1}\rightarrow F(w)=\frac{e^{-iw\pi}+1}{1-w^2}$$

---

80)

$$F(w)=\frac{2\sin w}{a+2w}$$

$$F(w)=\frac{i\left(\frac{e^{iw}-e^{-iw}}{2i}\right)}{a+iw}=\frac{1}{2}\frac{e^{iw}-e^{-iw}}{a+iw}$$

$$\frac{1}{2}\left[\frac{e^{iw}}{a+iw}\right]-\frac{1}{2}\left[\frac{e^{-iw}}{a+iw}\right]\Rightarrow\frac{1}{2}\left[e^{-a(x+1)}-e^{-a(x-1)}\right]=F^{-1}(w)$$

$$F^{-1}\left(\frac{2\sin w}{a+iw}\right)=\pi\left(e^{-a(x+1)}-e^{-a(x-1)}\right)$$

---

81)

$$F(w)=\frac{e^{-2iw}}{(3+iw)^3}$$

$$F(w)=\frac{1}{3+iw}\Rightarrow F^3(w)=\frac{-2}{(3+iw)^3}$$ مشتق سوم

$$F(w)=\frac{1}{(3+iw)^3}\Rightarrow F^{-1}\left(\frac{1}{(3+iw)^3}\right)=\frac{(-ix)^3e^{-3x}}{-2}=\frac{ix^3e^{-3x}}{-2}$$

$$F^{-1}\left(\frac{e^{-2iw}}{(3+iw)^3}\right)=\left(\frac{2(x-2)^3e^{-3(x-2)}}{-2}\right)2\pi$$

$$F^{-1}\left(\frac{e^{-2iw}}{(3+iw)^3}\right)=-(i-(x-2)^3e^{-3(x-2)})\pi$$

---

82)

$$F(x)=\frac{1}{iw+1-i}$$

$$F(w)=\int_{-\infty}^{\infty}F(x)e^{-iwx}dx$$: تبديل فوريه

$$F(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(w)e^{iwx}dw$$: تبديل وارون فوريه

$$F(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{iw+1-i}e^{iwx}.dw$$

$$F^{-1}\left(\frac{1}{1+i(w-1)}\right)=e^{ix}.F(x)=e^{ix}.e^{-x}.H(x)=e^{(ix-x)}.H(x)$$

---

83) $F(w)=\dfrac{e^{-2iw}}{-w^2+4iw}+3$

$$G(w)=\frac{1}{(iw+3)(iw+1)}=\frac{1}{2}\left(\frac{1}{iw+1}-\frac{1}{iw+3}\right)$$

$$G^{-1}\left\{\frac{1}{2}\left(\frac{1}{w+1}-\frac{1}{iw+3}\right)\right\}=\frac{e^{-x}-e^{-3x}}{2}H(x)$$

$$F^{-1}\left\{\frac{e^{-2iw}}{2}.G(w)\right\}=\frac{e^{-(x-2)}-e^{-3(x-2)}}{2}H(x-2)$$

---

87)

$$F(w)=\tan^{-1}\left(\frac{w}{a}\right)$$

$$F'(w)=\frac{\frac{1}{a}}{1+\left(\frac{w}{a}\right)^2}=\frac{a}{w^2+a^2}$$

$$F\{F(x)\}=F(w)$$

$$F^{-1}\{F'(w)\}=-2xF(x)\Rightarrow$$

$$\Rightarrow -2xF(x)=F^{-1}\left\{\frac{a}{w^2+a^2}\right\}\Rightarrow -2xF(x)=aF^{-1}\left\{\frac{1}{w^2+a^2}\right\}\Rightarrow -2xF(x)=a\times\frac{1}{2}e^{-a|x|}$$

$$\Rightarrow F(x)=\frac{2e^{-a|x|}}{2x}$$

---

88)

$$F(w)=\ln\left(\frac{w+b}{w+a}\right)$$

$$F(w)=\ln(w+b)-\ln(w+a)\Rightarrow F'(w)=\frac{1}{w+b}-\frac{1}{w+a}$$

$$F^{-1}\left\{\frac{1}{w}\right\}=F^{-1}\left\{\frac{i}{iw}\right\}=iH(x) \qquad ,F^{-1}\{F'(w)\}=-ixF(x)$$

$$\Rightarrow F^{-1}\left\{\frac{1}{w+b}-\frac{1}{w+a}\right\}=-ixF(x)\Rightarrow \left(ie^{-ibx}-ie^{-iax}\right)H(x)=-ixF(x)$$

$$\Rightarrow F(x)=\frac{1}{x}H(x)(e^{-iax}-e^{-ibx})$$

---

89)

$$F(w)=\frac{e^{-ikw}-1}{2\pi^2(a^2+w^2)}$$

$$F(w)=\frac{1}{2\pi^2}\left(\frac{e^{-ikw}}{a^2+w^2}-\frac{1}{a^2+w^2}\right)\Rightarrow F^{-1}(w)=\frac{1}{2\pi^2}F^{-1}\left\{\frac{e^{-ikw}}{a^2+w^2}-\frac{1}{a^2+w^2}\right\}$$

$$F^{-1}(w)=\frac{1}{2\pi^2}\left(\frac{1}{2a}e^{-a|x-k|}-\frac{1}{2a}e^{-a|x|}\right)\Rightarrow F(x)=\frac{1}{4a\pi^2}\left(e^{-a|x-k|}-e^{-a|x|}\right)$$

---

90)

$$F(w)=\frac{1}{w^4+4a^4}$$

$$w^4+4a^4=(w^2+2a^2)^2-4a^2w^2=(w^2+2a^2+2aw)(w^2+2a^2-2aw)=\left[(w-a)^2+a^2\right]\times\left[(w+a)^2+a^2\right]$$

$$\Rightarrow F(w)=\frac{1}{\left[(w+a)^2+a^2\right]\times\left[(w-a)^2+a^2\right]}=\frac{1}{4aw}\left[\frac{1}{(w-a)^2+a^2}-\frac{1}{(w+a)^2+a^2}\right]$$

$$\Rightarrow F^{-1}(w)=\frac{1}{4a}F^{-1}\left\{\frac{1}{w}\times\frac{1}{(w-a)^2+a^2}\right\}-\frac{1}{4a}F^{-1}\left\{\frac{1}{w}\times\frac{1}{(w+a)^2+a}\right\}$$

$$F^{-1}\left\{\frac{1}{(w-a)^2+a^2}\right\}=\frac{1}{2a}e^{iax}e^{-a|x|}\qquad ,F^{-1}\left\{\frac{1}{(w+a)^2+a^2}\right\}=\frac{1}{2a}e^{-iax}e^{-a|x|}$$

فرض: $F^{-1}\{F(w)\}=F(x)\Rightarrow F^{-1}\left\{\frac{F(w)}{w}\right\}=\int if(x)dx$

$$\Rightarrow F^{-1}\left\{\frac{1}{w}\times\frac{1}{(w+a)^2+a^2}\right\}=\int i\times\frac{1}{2a}e^{-iax}e^{-a|x|}dx=\begin{cases}\frac{-1}{a+ia}e^{-(ia+a)x}\times\frac{i}{2a} & x>0\\ \frac{1}{a-ia}e^{(a-ia)x}\times\frac{i}{2a} & x<0\end{cases}$$

$$\Rightarrow F^{-1}\left\{\frac{1}{w}\times\frac{1}{(w-a)^2+a^2}\right\}=\int i\times\frac{1}{2a}e^{iax}e^{-a|x|}dx=\begin{cases}\dfrac{-1}{a-ia}e^{(ia-a)x}\times\dfrac{i}{2a} & x>0\\ \dfrac{1}{a+ia}e^{(a+ia)x}\times\dfrac{i}{2a} & x<0\end{cases}$$

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91)

$$F(w)=\frac{8+4iw}{(1+iw)^2(3+iw)^2}$$

$$F(w)=\frac{2[(1+iw)+(3+iw)]}{(1+iw)^2(3+iw)^2}=\frac{2}{(1+iw)(3+iw)}\left[\frac{1}{3+iw}+\frac{1}{1+iw}\right]$$

$$F(w)=\left(\frac{1}{1+iw}-\frac{1}{3+iw}\right)\left(\frac{1}{1+iw}+\frac{1}{3+iw}\right)=\frac{1}{(1+iw)^2}-\frac{1}{(3+iw)^2}$$

$$F^{-1}(w)=F^{-1}\left\{\frac{1}{(1+iw)^2}-\frac{1}{(3+iw)^2}\right\}=F^{-1}\left\{\frac{1}{(1+iw)^2}\right\}-F^{-1}\left\{\frac{1}{(3+iw)^2}\right\}$$

$$\Rightarrow F^{-1}(w)=F(x)=xe^{-x}H(x)=xe^{-3x}H(x)$$

---

92)

$$F(x)=e^{-x}\sin\pi x \qquad\qquad x>0$$

$$F(w)=\int_0^{+\infty}e^{-x}\sin\pi x e^{-iwx}dx=\int_0^{+\infty}e^{-(1+iw)x}\sin 5dx=\left[-\frac{1}{5}e^{-(1+iw)x}\cos 5x-\frac{1}{2n}(1+iw)e^{-(1+iwx)}\right.$$

$$\left.\times\sin 5x\Big|_0^{\infty}-\frac{1}{2\pi}(1+iw)^2\int_0^{+\infty}e^{-(1+iw)x}\sin 5xdx\right]\Rightarrow\left[1+\frac{1}{25}(1+iw)^2\right]\int_0^{+\infty}e^{-(1+iw)x}\sin 5xdx=\frac{1}{5}$$

$$\Rightarrow F(w)=\frac{-1}{5+\frac{1}{5}(1+iw)^2}$$

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93)

$$F(x)=\frac{\sin 3x}{x^2+4x+20} \qquad\qquad x>0$$