$$F(x)=\frac{\frac{1}{2i}(e^{3ix}-e^{-3ix})}{(x+2)^2+16}=\frac{1}{2i}\times\frac{e^{3ix}}{(x+2)^2+16}-\frac{1}{2i}\times\frac{e^{-3ix}}{(x+2)^2+16}$$

$$\Rightarrow F(w)=\frac{1}{2i}F\left\{\frac{e^{3ix}}{(x+2)^2+16}\right\}-\frac{1}{2i}F\left\{\frac{e^{-3ix}}{(x+2)^2+16}\right\}=\frac{1}{2i}e^{-6i}F\left\{\frac{e^{3i(x+2)}}{(x+2)^2+16}\right\}-\frac{1}{2i}F\left\{\frac{e^{-3i(x+2)}}{(x+2)^2+16}\right\}$$

$$\Rightarrow F(w)=\frac{1}{2i}e^{-6i}e^{2iw}\times\frac{\pi}{4}e^{-4|w-3|}-\frac{1}{2i}e^{6i}e^{2iw}\times\frac{\pi}{4}e^{-4|w+3|}$$

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94) $F(x)=\frac{\sin x}{x}$ $x>0$ $F\{F(x)\}=F(w)$

$$2\pi F(-w)=F\{F(x)\}\Rightarrow 2\pi\frac{\sin w}{w}=F\{F(x)\}\Rightarrow\frac{\sin w}{w}=\frac{1}{2\pi}F\{F(x)\}$$

$$\Rightarrow F^{-1}\left\{\frac{\sin w}{w}\right\}=\frac{1}{2\pi}F(x)$$

$$\frac{\sin w}{w}=\frac{1}{2}\frac{e^{iw}-e^{-iw}}{iw}=\frac{1}{2}\times\frac{e^{iw}}{iw}-\frac{1}{2}\times\frac{e^{-iw}}{iw}$$

$$\Rightarrow F^{-1}\left\{\frac{\sin w}{w}\right\}=\frac{1}{2}[H(x+1)-H(x-1)]\Rightarrow F(w)=\pi[H(w+1)-H(w-1)]$$

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95)

$F(x)=\tan^{-1}x$ $x>0$

$F'(x)=\frac{1}{x^2+1}$ $F\left\{\frac{1}{x^2+}\right\}=\pi e^{-|w|}$ $F\{F'(x)\}=iwF(w)$

$$\Rightarrow \pi e^{-|w|}=iwF(w)\Rightarrow F(w)=\frac{\pi}{iw}e^{-|w|}$$

96)

$F(x)=J_0(ax)$ $x>0$

$y=J_0(x)=xy''+y'+xy=0\Rightarrow F\{xy''\}+F\{y'\}+F\{xy\}=0$ $F(w)=F\{y\}$

$$\Rightarrow iF'\{y''\}+iwF(w)+iF'(w)=0\Rightarrow i(-w^2F(w)')+iwF(w)+if(w)=0$$

$$\Rightarrow (w^2-1)F(w)=wF(w)=0\Rightarrow \frac{df}{F}+\frac{w}{w^2-1}dw=0\Rightarrow F=e^{-\int\frac{w}{w^2-1}dw}\Rightarrow F(w)=\frac{1}{\sqrt{w^2-1}}$$

$$F\{J_0(x)\}=\frac{1}{\sqrt{w^2-1}}\Rightarrow F\{J_0(ax)\}=\frac{1}{|a|}\times\frac{1}{\sqrt{\frac{w^2}{a^2}-1}}=\frac{1}{\sqrt{w^2-a^2}}$$

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97)

$$\to F(x)=1$$

$$F(x)=H(x), x>0 \qquad H(x)=\begin{cases}1 & x>0\\ 0 & x<0\end{cases}$$

$$F(F(x))=\int_0^\infty e^{-iwx}dx=\frac{1}{iw}$$

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98) $F(x)=e^{-ax}/x, x, a>0$

اگر : $F^{-1}\left(\frac{1}{a+iw}\right)=e^{-ax}H(x)$: می دانیم

$$a=0\to F^{-1}\left(\frac{1}{iw}\right)=H(x)\Rightarrow F^{-1}\left(\frac{1}{w}\right)=iH(x)$$

$$:2\pi F(-w)=F(f(x))\Rightarrow 2\pi i.H(-w)=F\left(\frac{1}{x}\right)$$

$$\to F\left(\frac{e^{-ax}}{x}\right)=2iH(ia-w)$$

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99) $F(w)=\frac{1}{\sqrt{a-w}}$ $\quad ,a>w>0$

$$F^{-1}(w)=\int_0^a \frac{1}{\sqrt{a-w}}e^{iwx}.dw$$

تغيير متغيير:

$a-w=\lambda \Rightarrow w=a-\lambda, dw=-d\lambda, w:0\to a \Rightarrow \lambda : a\to 0$

$$\Rightarrow F^{-1}(w)=\int_a^0 \frac{1}{\sqrt{\lambda}}e^{ix(a-\lambda)}(-d\lambda)=\int_0^a \frac{1}{\sqrt{2}}e^{ix(a-\lambda)}.d\lambda$$

$I_1 \qquad I_2$

$$=e^{iax}\left[\int_0^\infty \lambda^{-\frac{1}{2}}.e^{-ix\lambda}.d\lambda+\int_\infty^a \lambda^{-\frac{1}{2}}.e^{-i\lambda x}.d\lambda\right]$$

مى دانيم $\mu(n)=\int_0^\infty e^{-t}.t^{n-1}.dt, erFc(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u2}.du$

$I_1=\int_0^\infty \lambda^{-\frac{1}{2}}.e^{-ix\lambda}d\lambda$ تغيير متغير $i\lambda x=t\to \lambda=\frac{t}{ix}, d\lambda=\frac{dt}{ix}$

$$I_1=\int_0^\infty\left(\frac{t}{ix}\right)^{-\frac{1}{2}}.e^{-t}\left(\frac{d\lambda}{ix}\right)=\frac{\sqrt{ix}}{ix}\int_0^\infty t^{-\frac{1}{2}}.e^{-t}.dt=(ix)^{-\frac{1}{2}}.\mu\left(\frac{1}{2}\right)$$

$I_2=\frac{-\sqrt{\pi}}{2}\int_a^\infty \lambda^{-\frac{1}{2}}.e^{-i\lambda x}.d\lambda$ اگر : $i\lambda x=u^2\to du=\frac{ix.d\lambda}{\sqrt{ix}}(\lambda)^{-\frac{1}{2}}$

$$\Rightarrow I_2=-\left[\int_a^\infty \frac{ix}{2\sqrt{ix}}.(\lambda)^{-\frac{1}{2}}.e^{-i\lambda x}.d\lambda\right]\frac{2\sqrt{ix}\sqrt{\pi}}{ix2}=-2(ix)^{-\frac{1}{2}}.erFC(a).\frac{\sqrt{5}}{2}$$

$$\to F^{-1}(w)=(I_1+I_2)e=\frac{\sqrt{\pi}e^{iax}}{\sqrt{ix}}[1-erFC(a)]$$

90)

$$F(x)=\frac{1}{\sqrt{a^2-w^2}} \qquad ,a>w>0$$

می دانیم: $F\left(J_0(ac)\right)=\frac{1}{\sqrt{w^2-a^2}} \longrightarrow \times\frac{i}{i} \qquad F(J_0(ax))=\frac{i}{\sqrt{a^2-w^2}}$

$$\xrightarrow{\pi=1} \quad F^{-1}\left(\frac{i}{\sqrt{a^2-w^2}}\right)=J_0(ax)\Rightarrow F^{-1}(w)=\frac{J_0(ax)}{i}$$

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101) $F(w)=\frac{1}{\sqrt{w^2+a^2}}$

می دانیم: $F(y_0(bx))=\frac{1}{\sqrt{w^2-b^2}} \longrightarrow F^{-1} \qquad J_0(bx)=F^{-1}\left(\frac{1}{\sqrt{w^2-b^2}}\right)$

معرفی می کنیم: $b=ia\Rightarrow F^{-1}\left(\frac{1}{\sqrt{w^2+a^2}}\right)=J_0(iax)$

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102) $F(x)=x^{-m} \qquad 0<x<\infty,0<m<1$

می دانیم $L(F(x))=\int_0^\infty F(x).e^{-px}dx$

$$F(f(x))=\int_0^\infty F(x).e^{-iwx}.dx$$

و می دانیم: $L(t^\alpha)=\mu(\alpha+1)/P^{\alpha+1}\rightarrow L(x)^{-m}=\frac{\mu(1-m)}{P(1-m)}$

$$p\rightarrow iw\Rightarrow L\rightarrow F\Rightarrow F(x)^{-m}=\frac{\mu(1-m)}{(+iw)^{(1-m)}}$$

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103) $F\left(J_0(ax)\right)\sin\left(\beta x\right) \qquad ,x>0$

می دانیم $\sin\theta=\frac{1}{2i}(e^{i\theta}-e^{-i\theta})\Rightarrow \sin\left(\beta x\right)=\frac{1}{2i}\left(e^{i\beta x}-e^{-i\beta x}\right)$

$$\Rightarrow F(x)=\frac{1}{2i}\left[e^{i\beta x}.J(ax)-e^{-i\beta x}.y(ax)\right], F(J_0(ax))=\frac{1}{\sqrt{w^2-a^2}}$$

$$\Rightarrow F(e^{i\beta x}.J_0(ax))=\frac{1}{\sqrt{(w-i\beta)^2-a^2}}, F(e^{-i\beta x}.J_0(ax))=\frac{1}{\sqrt{(w+i\beta)^2-a^2}}$$

$$\Rightarrow F(f(x))=2i\left(\frac{1}{\sqrt{(w-i\beta)^2-a^2}}+\frac{1}{\sqrt{(w+i\beta)^2-a^2}}\right)$$

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104) $F(x)S(x-x_0)$

مي دانیم $8(x)=\begin{cases}1/4 & 0\le x\le 4\\ 0 & x\ge 4\end{cases}$

$$\Rightarrow F(S(x))=\int_0^4\frac{1}{4}.e^{-iwx}.dx=\frac{1}{4}\int_0^4 e.dx=\frac{1}{4iw}\left[-e^{-iwx}\right]_0^4=\frac{1}{iw4}\left[1-e^{-iw4}\right]$$

اگر $F(x)\to F(x-x_0)\Rightarrow F(w)\to e^{-iwx}F(w)\Rightarrow F(f(x))=\frac{e^{-iwx}}{iw4}\left[1-e^{-iw4}\right]$

$F(w)=\frac{1}{w}\ln\left(\frac{w+b}{w+a}\right)$: نا معلوم

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106) $F(x)=\frac{1}{\sqrt{x}} \qquad ,x>0$

$$F(x) = x^{-1/2}, 0 < x < \infty, 0 < \frac{1}{2} < 1$$

با توجه به سوال 102: $F(f(x)) = \frac{\mu(1-1/2)}{iw^{(1-1/2)}} = \frac{\sqrt{\pi}}{\sqrt{iw}} = \sqrt{\frac{\pi}{iw}}$

107) $F(x) = x^n . \sin(\beta x) . e^{-ax}$

معرفی می کنیم:

$$g(x) = \sin(\beta x) . e^{-ax} = \frac{1}{2i}(e^{i\beta x} - e^{-i\beta x})e^{-ax} = \frac{1}{2i}\left(e^{(i\beta - a)x} - e^{-(i\beta + a)x}\right)$$

$$\Rightarrow F(g(x)) = \frac{1}{2i}\left(\frac{1}{i\beta - iw - a} - \frac{1}{i\beta + iw + a}\right) = \frac{-1}{2i}\left(\frac{1}{iw + i\beta + a} - \frac{1}{iw - i\beta + a}\right)$$

$$\Rightarrow F(f(x)) = \frac{1}{2i} \times \left(\frac{-1}{i}\right)^n \left[\frac{1}{iw + i\beta + a} - \frac{1}{iw - i\beta + a}\right]^{(n)}$$

$$\Rightarrow F(f(x)) = \frac{1}{2}\left(\frac{-1}{i}\right)^{n+1}\left[\frac{ni(-i)^n}{(iw + i\beta + a)^{n+1}} - \frac{ni(-i)^n}{(iw - i\beta + a)^{n+1}}\right] = \frac{-1}{2i}\left(\frac{1}{(iw - i\beta + a)^{n+1}} - \frac{1}{(iw - i\beta + a)^{n+1}}\right)$$