$$-\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2-4}e^{x}\cos\tfrac{n\pi}{2}x\right\}_{-1}^{1}+\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2+4}e^{-x}\cos\tfrac{n\pi}{2}x\right\}_{-1}^{1}$$

$$-\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2-4}e^{x-2}\cos\tfrac{n\pi}{2}x\right\}_{1}^{2}+\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2+4}e^{-x+2}\cos\tfrac{n\pi}{2}x\right\}_{1}^{2}$$

$$=\left\{-\tfrac{4}{n\pi}(-1)^n\right\}+\left\{\tfrac{4}{n\pi}(-1)^n-\tfrac{4}{n\pi}\cos\tfrac{n\pi}{2}\right\}-\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2-4}\left(e\cos\tfrac{n\pi}{2}(-1)^n\right)\right\}$$

$$+\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2+4}\left(e^{-1}\cos\tfrac{n\pi}{2}-(-1)^n\right)\right\}-\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2-4}\left(e\cos\tfrac{n\pi}{2}-e^{-1}\cos\tfrac{n\pi}{2}\right)\right\}$$

$$+\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2+4}\left(e^{-1}\cos\tfrac{n\pi}{2}-e\cos\tfrac{n\pi}{2}\right)\right\}-\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2-4}\left((-1)^n-e^{-1}\cos\tfrac{n\pi}{2}\right)\right\}$$

$$+\tfrac{1}{2}\left\{\frac{n\pi}{n^2\pi^2+4}\left((-1)^n+e\cos\tfrac{n\pi}{2}\right)\right\}$$

$$=\tfrac{1}{2}\left\{\left(-e\cos\tfrac{n\pi}{2}+(-1)^n\right)\left(\frac{n\pi}{n^2\pi^2+4}+\frac{n\pi}{n^2\pi^2-4}\right)\right\}$$

$$+\tfrac{1}{2}\left\{\left(e^{-1}\cos\tfrac{n\pi}{2}-(-1)^n\right)\left(\frac{n\pi}{n^2\pi^2+4}+\frac{n\pi}{n^2\pi^2-4}\right)\right\}$$

$$+\tfrac{1}{2}\left\{\left(e^{-1}\cos\tfrac{n\pi}{2}-e\cos\tfrac{n\pi}{2}\right)\left(\frac{n\pi}{n^2\pi^2+4}+\frac{n\pi}{n^2\pi^2-4}\right)\right\}$$

$$b_n=\left\{\left(\frac{n\pi}{n^2\pi^2+4}+\frac{n\pi}{n^2\pi^2-4}\right)\left(e^{-1}\cos\tfrac{n\pi}{2}-e\cos\tfrac{n\pi}{2}\right)\right\}$$

دامنه مركب فوريه:

$F(x)=|x|+\sinh x \qquad -1<x<1$

$F(x)=A_{\circ}+\sum A_n\cos\left(\tfrac{n\pi}{l}x-\beta_n\right)$

$\beta_n=Arctg\left(\tfrac{b_n}{a_n}\right)$

$A_n=\sqrt{a_n^2+b_n^2}$

سری مختلط نمايي فوريه:

$F(x)=|x|+\sinh x \qquad -1<x<1$

$$F(x) = C_\circ + \sum c_n e^{\frac{in\pi}{l}x}$$

$$C_n = \frac{1}{2l}\int_c^{c+2l} F(x) e^{-\frac{in\pi}{l}x}dx, C_n = \frac{1}{2}(a_n - ib_n)$$

$$C_n = \frac{1}{2}\left\{\frac{2\left((-1)^n - 1\right)}{n^2\pi^2} - i(-1)^n(e - e^{-1})\left(\frac{n\pi}{n^2\pi^2+1} - \frac{n\pi}{n^2\pi^2-1}\right)\right\}$$

اتحاد پارسوال

$$\frac{1}{l}\int_c^{c+2l} F(x)^2 dx = \frac{a_0^2}{2} + \sum a_n^2 + b_n^2 \qquad l = 1$$

$$\int_{-1}^{1}\left(|x| + \sinh x\right)^2 dx = \int_{-1}^{1}\left(x^2 + 2|x|\sinh x + \sin^2 hx\right)dx$$

$$= \frac{x^3}{3}\Big|_{-1}^{1} + \frac{1}{4}\left(\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} - 2x\right)\Big|_{-1}^{1}$$

$$= \frac{2}{3} + \frac{1}{8}\left((e^2 - e^{-2} - 4) - (e^{-2} - e^2 + 4)\right)$$

$$= \frac{2}{3} + \frac{1}{8}\left(2e^2 - 2e^{-2} - 8\right) = \frac{2}{3} - 1 + \frac{1}{4}\left(e^2 - e^{-2}\right) = -\frac{1}{3} + \frac{1}{4}\left(e^2 - e^{-2}\right)$$

$$\frac{1}{2} + \sum\left(\frac{2\left((-1)^2 - 1\right)}{n^2\pi^2}\right)^2 + \left\{\left((-1)^n(e - e^{-1})\left(\frac{n\pi}{n^2\pi^2+1} - \frac{n\pi}{n^2\pi^2-1}\right)\right)^2\right\}$$

$$\Rightarrow \sum\left(\frac{2\left((-1)^n - 1\right)}{n^2\pi^2}\right)^2 + \left((-1)^n(e - e^{-1})\left(\frac{n\pi}{n^2\pi^2+1} - \frac{n\pi}{n^2\pi^2-1}\right)\right)^2 = \frac{5}{6} + \frac{1}{4}(e^2 - e^{-2})$$

S.F (7

$x^2 + \sin^3 \pi x \qquad -1 < x < 1 \qquad l = 1$

$$a_0 = \int_{-1}^{1}\left(x^2 + \sin^3 \pi x\right)dx = \int_{-1}^{1} x^2 dx + \int_{-1}^{1}\sin^3 \pi x dx \qquad \frac{a_0}{2} = \frac{1}{3}$$

$$a = \int_{-1}^{1}(x^2 + \sin^3 \pi x)\cos\frac{n\pi}{1}xdx = \int_{-1}^{1} x^2 \cos n\pi x dx + \int_{-1}^{1}\sin^3 \pi x \cos n\pi x dx$$

x^2	$\cos n\pi x$
	$\frac{1}{n\pi}\sin n\pi x$
$2x$	$-\frac{1}{n^2\pi^2}\cos n\pi x$
2	$-\frac{1}{n^3\rho^3}\sin n\pi x$
0	

$$=\left\{\frac{x^2}{n\pi}\sin n\pi x+\frac{2x}{n^2\pi^2}\cos n\pi x-\frac{2}{n^3\pi^3}\sin n\pi x\right\}$$

$$\frac{2}{n^2\pi^2}(-1)^n+\frac{2}{n^2\pi^2}(-1)^n=\frac{4}{n^2\pi^2}(-1)^n$$

$$a_n=\frac{4}{n^2\pi^2}(-1)^n$$

$$b_n=\int_{-1}^{1}\left(x^2+\sin^3\pi x\right)\sin n\pi x\,dx=\int_{-1}^{1}x^2\sin n\pi x\,dx+\int_{-1}^{1}\sin^3\pi x\sin n\pi x\,dx$$

فـرد

$$=-\int_{-1}^{1}\frac{1}{4}\sin 3x\sin n\pi x\,dx+\frac{3}{4}\int_{-1}^{1}\sin x\sin n\pi x\,dx$$

$$=\frac{1}{8}\int_{-1}^{1}\left(\cos(3+n\pi)x-\cos(3-n\pi)x\right)dx-\frac{3}{8}\int_{-1}^{1}\left(\cos(1+n\pi)x-\cos(1-n\pi)x\right)dx$$

$$=\frac{1}{4}\left\{\frac{1}{3+n\pi}\sin(3+n\pi)x-\frac{1}{3-n\pi}\sin(3-n\pi)x\right\}_0^1$$
$$-\frac{3}{4}\left\{\frac{1}{1+n\pi}\sin(1+n\pi)x-\frac{1}{1-n\pi}\sin(1-n\pi)x\right\}_0^1$$

$$\sin(3-n\pi)x=\sin(3-n\pi+2n\pi)$$
$$=\sin(\backepsilon+n\pi)$$

$$=\frac{1}{4}\left\{\sin(3+n\pi)x\left(\frac{1}{3+n\pi}-\frac{1}{3-n\pi}\right)\right\}_0^1-\frac{3}{4}\left\{\sin(1+n\pi)x\left(\frac{1}{1+n\pi}-\frac{1}{1-n\pi}\right)\right\}_0^1$$

$$b_n=\frac{1}{4}\left(\frac{-2n\pi}{9-n^2\pi^2}\right)(-1)^n\sin 3-\frac{3}{4}\left(\frac{(-2n\pi)}{1-n\pi}\right)(-1)^n\sin 1$$

F.C.S (7

$x^2+\sin^3\pi x \qquad -1<x<1$

$$F(x)=(x-1)^2+\sin^3\pi(x-1)$$

$$F(x)=\begin{cases}(x-1)^2+\sin^3\pi(x-1) & 0<x<2\\ (x+1)^2-\sin^3\pi(x+1) & -2<x<0\end{cases}\qquad l=2$$

$$F(x) = a_{0/2} + \sum a_n \cos {}^{n\pi}\!/_{e}\, x$$

$$a_n = {}^{1}\!/_{l} \int_{c}^{c+21} F(x) \cos {}^{n\pi}\!/_{e}\, x dx$$

$$= {}^{1}\!/_{2} \int_{-2}^{0} (cx+1)^2 - \sin^3 \pi (x+1) \cos {}^{n\pi}\!/_{l}\, x + {}^{1}\!/_{2} \int_{0}^{2} \left((x-1)^2 + \sin^3 \pi (x-1)\right) \cos {}^{n\pi}\!/_{l}\, x dx$$

F.S.S (7

$$F(x) = x^2 + \sin^3 \pi x$$

$0 < x < 2$

$$F(x) = \begin{cases} (x-1)^2 + \sin^3 \pi (x-1) \\ -(x+1)^2 + \sin^3 \pi (x+1) \end{cases}$$

$$F(x) = \sum b_n \sin {}^{n\pi}\!/_{2}\, x$$

$$b_n = {}^{1}\!/_{e} \int_{c}^{c+21} F(x) \sin {}^{n\pi}\!/_{2}\, x$$

$$= {}^{1}\!/_{2} \int_{0}^{2} \left((x-1)^2 + \sin^3 \pi (x-1)\right) \sin {}^{n\pi}\!/_{2}\, x dx + {}^{1}\!/_{2} \int_{-2}^{0} \left(-(x+1)^2 + \sin^3 \pi (x+1)\right) \sin {}^{n\pi}\!/_{2}\, x dx$$

7) دامنه ی مرکب فوریه :

$$F(x) = x^2 + \sin^3 \pi x \qquad -1 < x < 1$$

$$F(x) = A_\circ + \sum A_n \cos\left({}^{n\pi}\!/_{e}\, x - \beta\right)$$

$$A_\circ = a_{0/2}$$

$$\beta_n = Arctg\, {}^{b_n}\!/_{a_n}$$

$$A_n = \sqrt{a_n^2 + b_n^2}$$

سری مختلط نمایی فوریه :

$$F(x) = C_{\circ} + \sum c_n e^{\frac{in\pi}{l}x}$$

$$c_0 = a_{0}/_{2}$$

$$C_n = 1/_{l}\,(a_n - ib_n) \qquad , C_n = 1/_{21} \int_{c}^{c+21} F(x) e^{-\frac{in\pi}{l}x} dx$$

$$l = 1$$

قضیه دریکله :

$$x = -1 \qquad F(-1^{+}) = 1 \quad F(-1^{-}) \qquad F(-1) = \frac{|+|}{2} = 1$$

سری فوریه: $a_{0}/_{2} + \sum_{n=1}^{\infty} a \cos n\pi/_{l}\, x + b \sin n\pi/_{l}\, x$

$$: 1/_{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n (-1)^n}{n^2\pi^2} + 0 = 1/_{3} + 4/_{\pi^2} \sum_{n=1}^{\infty} 1/_{n^2}$$

$$1/_{3} + 4/_{\pi^2} \sum_{n=1}^{\infty} 1/_{n^2} = 1 \qquad \sum_{n=1}^{\infty} 1/_{n^2} = \pi^2/_{6}$$

$$x = 1 \qquad F(1^{+}) = 1 \quad F(1^{-}) = 1 \qquad F(1) = \frac{1+1}{2} = 1$$

سری فوریه: $a_{0}/_{2} + \sum a_n \cos n\pi/_{e}\, x + b_n \sin n\pi/_{l}\, x$

$$1/_{3} + \sum \frac{4(-1)^n (-1)^n}{n^2\pi^2} + 0 = 1/_{3} + 4/_{\pi^2} \sum_{n=1}^{\infty} 1/_{x^2}$$

$$\sum 1/_{n^2} = x^2/_{6}$$

S.F (8

$$F(x) = e^{-x} \qquad 0 < x < l$$

$$a_0 = \frac{1}{l}\int_c^{c+21} F(x)dx = \frac{2}{l}\int_0^l e^{-x}dx = \frac{2}{l}\left\{-e^{-x}\right\}_0^l$$

$$a_{0/2} = \frac{1}{e}(-e^{-e}+1)$$

$$a_n = \frac{2}{l}\int_0^l e^{-x}\cos\frac{2n\pi}{l}xdx$$

e^{-x}	$\cos\frac{2n\pi}{l}x$
$-e^{-x}$	$\frac{l}{2n\pi}\sin\frac{2n\pi}{l}x$
e^{-x}	$-\frac{l^2}{4n^2\pi^2}\cos\frac{2n\pi}{l}x$

$$I = \int_0^e e^{-x}\cos\frac{2n\pi}{l}xdx$$

$$I = \frac{l}{2n\pi}e^{-x}\sin\frac{2n\pi}{l}x - \frac{e^2}{4n^2\pi^2}I$$

$$I = \frac{1}{1+\frac{l^2}{4n^2\pi^2}}\left\{\frac{l}{2n\pi}e^{-x}\sin\frac{2n\pi}{l}x\right\}$$

$$a_n = \frac{4n^2\pi^2}{4n^2\pi^2+l}\left\{\frac{1}{n\pi}e^{-x}\sin\frac{2n\pi}{l}x\right\}_0^l = 0$$

$$b_n = \frac{2}{e}\int_0^l e^{-x}\sin\frac{2n\pi}{l}xdx$$

e^{-x}	$\sin\frac{2n\pi}{l}x$
$-e^{-x}$	$-\frac{l}{2n\pi}\cos\frac{2n\pi}{l}x$
e^{-x}	$-\frac{l^2}{(2n\pi)^2}\sin\frac{2n\pi}{e}x$

$$I = \int_0^e e^{-x}\sin\frac{2n\pi}{e}xdx$$

$$I = -\frac{e}{2n\pi}\cos\frac{2n\pi}{l}x - \frac{l^2}{4n^2\pi^2}\sin\frac{2n\pi}{l}x$$

$$I = \frac{1}{1+\frac{l^2}{4n^2\pi^2}}\left\{-\frac{l}{2n\pi}e^{-x}\cos\frac{2n\pi}{l}x\right\}_0^e$$

$$b_n = \frac{4n^2\pi^2}{4n^2\pi^2+l}\left\{-\frac{1}{n\pi}e^{-e}+\frac{1}{n\pi}\right\} = \frac{4n\pi}{4n^2\pi^2+l^2}\left\{1-e^{-l}\right\}$$

F.C.S

$$F(x)=\begin{cases} e^{-x} & 0<x<l \\ e^{x} & -l<x<0 \end{cases}$$

$$a_{0/2}=2/l\,(-e^{-l}+1)$$

$$a_n=1/l\int_{-l}^{0}e^{x}\cos\frac{n\pi}{l}x\,dx+1/l\int_{0}^{l}e^{-x}\cos\frac{n\pi}{l}x\,dx$$

e^{x}	$\cos\frac{n\pi}{l}x$
ex	$\frac{l}{n\pi}\sin\frac{n\pi}{l}x$
e^{x}	$-\frac{l^2}{n^2\pi^2}\cos\frac{n\pi}{l}x$

e^{-x}	$\cos\frac{n\pi}{l}x$
$-e^{-x}$	$\frac{l}{n\pi}\sin\frac{n\pi}{l}x$
e^{-x}	$-\frac{l^2}{n^2\pi^2}\cos\frac{n\pi}{l}x$

$$I=\int_{-1}^{0}e^{x}\cos\frac{n\pi}{l}x\,dx \qquad I=\int_{0}^{l}e^{-x}\cos\frac{n\pi}{l}x\,dx$$

$$I=\frac{1}{1-\frac{l^2}{n^2\pi^2}}\left\{\frac{le^{x}}{n\pi}\sin\frac{n\pi}{l}x\right\}_{-l}^{0} \qquad I=\frac{1}{1+\frac{l^2}{n^2\pi^2}}\left\{\frac{l}{n\pi}e^{-x}\sin\frac{n\pi}{l}x\right\}_{0}^{l}$$

$$a_n=\frac{n\pi}{n^2\pi^2-l^2}\left\{e^{x}\sin\frac{n\pi}{l}x\right\}_{-l}^{0}+\frac{n\pi}{n^2\pi^2+l^2}\left\{e^{-x}\sin\frac{n\pi}{l}x\right\}_{0}^{e}=0$$

F.S.S

$$F(x)=\begin{cases} e^{-x} & 0<x<l \\ -e^{x} & -e<x<0 \end{cases}$$

$$F(x)=\sum b_n \sin\frac{n\pi}{l}x$$

$$b_n = {}^{1}\!/_{e}\int_{c}^{c+21} F(x)\sin\frac{n\pi}{l}xdx$$

$$= {}^{1}\!/_{e}\int_{-l}^{0} -e^{x}\sin\frac{n\pi}{l}xdx + {}^{l}\!/_{e}\int_{0}^{e} e^{-x}\sin\frac{n\pi}{l}xdx$$

8) دامنه مرکب فوریه :

$$F(x)=e^{-x} \qquad 0<x<l \qquad l=\frac{l}{2}$$

$$F(x)=A_{\circ}+\sum A_n\cos\left(\frac{n\pi}{l}x-\beta_n\right) \qquad A_{\circ}=a_{0}/_{2}$$

$$\beta_n = tg^{-1}\, {b_n}/{a_n} = tg^{-1} b_{n}/_{0} = \pi/2$$

$$A_n=\sqrt{a_n^2+b_n^2}=\sqrt{b_n^2}=b_n$$

مختلط نمایی فوریه:

$$F(x)=e^{-x} \qquad 0<x<l$$

$$F(x)=c_{\circ}+\sum_{n=1}^{\infty}c_n e^{\frac{in\pi}{l}x} \qquad c_{\circ}=a_{0}/_{2}$$

$$c_n = {}^{1}\!/_{2l}\int F(x)e^{-\frac{in\pi}{l}x}dx = {}^{1}\!/_{l}\int_0^l e^{-x}e^{-\frac{2in\pi}{l}x}dx = {}^{1}\!/_{l}\int_0^l e^{(-1-\frac{2in\pi}{l})x}dx$$

$$c_n = {}^{1}\!/_{l}\left(\frac{1}{-1-\frac{2in\pi}{l}}\right)\left(e^{-l-2in\pi}-1\right)$$

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8) اتحاد پارسوال:

$$F(x)=e^{-x} \qquad 0<x<e$$

$$\frac{1}{e}\int F^2(x)dx=\frac{a_0^2}{2}+\sum a_n^2+b_n^2$$

$$\frac{1}{e}\int_c^{c+21}F^2(x)dx=\frac{2}{e}\int_0^e e^{-2x}dx=\frac{2}{l}\left\{-\frac{1}{2}e^{-2x}\right\}_0^l=-\frac{1}{e}\left\{e^{-2l}-1\right\}$$

$$=\frac{1}{e}\left\{1-e^{2l}\right\}$$

$$\frac{a_0^2}{2}+\sum a_n^2+b_n^2=\frac{1}{e^2}(1-e^{-e})^2+\sum\left(1-e^{-l}\right)^2\left(\frac{4n\pi}{l^2+4n^2\pi^2}\right)^2$$

$$\Rightarrow \frac{1}{e}(1-e^{-2l})=\frac{1}{e}(1-e^{-l})^2+\sum(1-e^{-l})^2\left(\frac{4n\pi}{l^2+4n^2\pi^2}\right)^2$$

$$\frac{\left(1-e^{-2l}\right)}{l(1-e^{-l})^2}=\frac{1}{l^2}+\sum\frac{16n^2\pi^2}{(l^2+4n^2\pi^2)^2}$$

سوال 9

$$F(x)=x(1-x)^2 \qquad 0\le x\le 1 \qquad 2l=1 \qquad l=\frac{1}{2}$$

$$a_0=2\int_0^1\left(x+x^3-2x\right)dx=2\left(\frac{x^2}{2}+\frac{x^4}{4}-\frac{2}{3}x\right)_0^1=\frac{1}{6}$$

$$a=2\int_0^1\left(x\cos 2nx\pi+x^3\cos 2n\pi x-2x^2\cos 2n\pi x\right)dx$$

$$=2\left\{\frac{x\sin 2n\pi x}{2n\pi}+\frac{\cos 2n\pi x}{4n^2\pi^2}+\frac{x^3\sin 2n\pi x}{2n\pi}+\frac{3x^2\cos 2n\pi x}{4x^2\pi^2}-\frac{6x\sin 2n\pi x}{8n^3\pi^3}-\frac{\cos 2n\pi x}{16n^4\pi^4}\right.$$

$$\left.-2\left(\frac{x^2}{2n\pi}\sin 2n\pi x+\frac{2x\cos 2n\pi x}{4n^2\pi^2}-\frac{2\sin 2n\pi x}{8n^3\pi^3}\right)\right\}_0^1=\frac{-1}{2n^2\pi^2}$$

$$b_n=2\int_0^1\left(x\sin 2n\pi x+x^3\sin 2n\pi x-2x^2\sin 2n\pi x\right)dx$$

$$=2\left\{\frac{-x\cos 2n\pi x}{2n\pi}+\frac{\sin 2n\pi x}{4n^2\pi^2}-\frac{x^3\cos 2n\pi x}{2n\pi}+\frac{3x^2\sin 2n\pi x}{4n^2\pi^2}+\frac{6x\cos 2n\pi x}{8n^3\pi^3}-\frac{6\sin 2n\pi x}{16n^4\pi^4}\right.$$

$$\left.+\frac{2x^2\cos 2n\pi x}{2n\pi}-\frac{4x\sin 2n\pi x}{4n^2\pi^2}-\frac{4\cos 2n\pi x}{8n^3\pi^3}\right\}_0^1=\frac{3}{2n^3\pi^3}$$

سری فوریه :

$$F(x)=\frac{1}{12}+\sum_{n=1}^{\infty}-\frac{1}{2n^2\pi^2}\cos 2n\pi x+\frac{3}{2n^3\pi^3}\sin 2n\pi x$$

$$C_n=\frac{1}{2}(a_n-ib_n)=\frac{1}{4}\left(\frac{-1}{n^2\pi^2}+\frac{i3}{n^3\pi^3}\right)$$

$n\neq 0$

$$C_0=a_{0/2}=\frac{1}{12}$$

سری نمايي:

$$F(x)=\frac{1}{12}+\frac{1}{4}\sum_{\substack{n=-\infty\\ n\neq 0}}^{n=+\infty}\left(-\frac{1}{n^2\pi^2}+\frac{i3}{n^3\pi^3}\right)e^{2n\pi i x}$$

$$A_n=\sqrt{a_n^2+b_n^2}=\frac{1}{2n^3\pi^3}\sqrt{9+n^2\pi^2}$$

$$A_0=a_{0/2}=\frac{1}{12}$$

$$B_n=\tan^{-1}\left(b_n/a_n\right)=\tan^{-1}\left(-3/n\pi\right)$$

دامنه $F(x)=\frac{1}{12}+\sum_{n=1}^{\infty}A_n\cos(2n\pi x-\beta_n)$

مرکب:

سری کسینوسی

$$a_0=\frac{1}{6}$$

$$n\Rightarrow n/2$$

$$a_n=2\left\{\frac{x\sin n\pi x}{n\pi}+\frac{\cos n\pi x}{n^2\pi^2}+\frac{x^3\sin n\pi x}{n\pi}+\frac{3x^2}{n^2\pi^2}\cos n\pi x-\frac{6x\sin n\pi x}{n^3\pi^3}-\frac{6\cos n\pi x}{n^4\pi^4}\right.$$

$$\left.-2\left(\frac{x^2\sin n\pi x}{n\pi}+\frac{2x\cos n\pi x}{n^2\pi^2}-\frac{2\sin n\pi x}{n^3\pi^3}\right)\right\}_0^1=\frac{12(-1)^{n+1}}{n^4\pi^4}$$