سری کسینوسی

$$F(x)=\frac{1}{12}+\sum\frac{12(-1)^n}{n^4\pi^4}\cos n\pi x$$

$$_{n\to n/2}b_n=2\left\{\frac{-x\cos n\pi x}{n\pi}+\frac{\sin n\pi x}{n^2\pi^2}-\frac{x^3\cos n\pi x}{n\pi}+\frac{3x^2\sin n\pi x}{n^2\pi^2}+\frac{6\cos n\pi x}{n^3\pi^3}-\frac{6\sin n\pi x}{n^4\pi^4}+\frac{2x^2\cos n\pi x}{n\pi}\right.$$

$$\left.-\frac{4x\sin n\pi x}{n^2\pi^2}-\frac{4\cos n\pi x}{n^3\pi^3}\right\}_0^1=\frac{4(-1)^n}{n^3\pi^3}$$

$F(x)=\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^3\pi^3}\sin n\pi x$ سری سینوسی

قضیه دیریکله :

$x=0$

$\lim F(x)=0$

$x\to 0^+$

$\lim F(x)=0$

$x\to 0^-$

نقطه $\frac{0+0}{2}\times\frac{1}{12}+\sum_{n=1}^{\infty}\frac{1}{2n^2\pi^2}\to \pi^2/6=\sum_{n=1}^{\infty}\frac{1}{n^2}$

ناپیوسته :

$x=1/2$

$F(1/2)=\frac{1}{8}$

$$\frac{1}{8}=\frac{1}{12}+\sum_{n=1}^{\infty}\frac{(-1)}{2n^2\pi^2}\to\frac{\pi^2}{12}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$$

سوال 10)

$F(x)=x+\sin x$ $\quad -\pi < x <$

$$2\pi = 2l \Rightarrow l = \pi$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\sin x)dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\sin x)\cos nx dx = 0$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(x\cos nx + \sin n \cos nx\right)dx = 0$$

$(I_1)=0 \qquad (I_2)$

$$I_2 = \int_{-\pi}^{\pi}\frac{1}{2}\left(\sin 2x + \sin x\right)dx = -\frac{1}{4}\cos 2x\Big|_{-\pi}^{\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\sin n)\sin nx dx = \int_{-\pi}^{\pi}\sin nx dx + \int_{-\pi}^{\pi}\sin x \sin x dx = \frac{-2\pi(-1)^n}{n}$$

$I_1 \qquad I_2$

$+x$	$\sin nx$
-1	$-\frac{\cos nx}{n}$
$+0$	$-\frac{\sin nx}{n^2}$

$$\Rightarrow \frac{-x\cos n}{n} + \frac{\sin nx}{n^2}\Big|_{-\pi}^{\pi} \Rightarrow I_1 = \frac{-\pi(-1)^n}{n} - \frac{\pi(-1)^n}{n} = \frac{-2\pi(-1)^n}{n}$$

$$I_2 = -\frac{1}{2}\int_{-\pi}^{\pi}\left[\cos(n+1) - \cos(1-n)\right]x dx = -\frac{1}{2}\left\{\frac{\sin(n+1)n}{n+1} - \frac{\sin(x-1)n}{n-1}\right\}_{-\pi}^{\pi} = 0$$

برای $n = 1 \Rightarrow b_1 = \int \frac{1}{2}\left(1+\cos 2x\right) = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)\Big|_{-\pi}^{\pi} = \pi$

برای $n = -1 \rightarrow b_1 = -\pi$

$$b_n = \begin{cases} \frac{-2}{n}(-1)^n & n \neq \pm 1 \\ 3 & n = 1 \\ -3 & n = -1 \end{cases} \qquad a_n = \begin{cases} a_n = 0 & n \neq \pm 1 \\ a_1 = a_{-1} = 0 \end{cases}$$

سری فوریه: $F(x) = \sum_{n=2}^{\infty}\left(\frac{-2}{\pi}(-1)^2\right)\sin nx + 3\sin x$

$$C_n = \frac{1}{2}\left(0 - i\left(\frac{-2}{\pi}(-1)^n\right)\right) = \frac{i}{n}(-1)^n$$

$n \neq 0$

$n \neq \pm 1$ $\qquad n = 0 \rightarrow C_0 = a_0/2 = 0$

$C_1 = \frac{1}{2}(0-3i) = -\frac{3}{2}i \qquad , C_{-1} = \frac{1}{2}(0-3i) = \frac{3}{2}i$

سری نمایی فوریه :

$$F(x) = \sum_{\substack{n=-\infty \\ n\neq 0 \\ n \neq \pm 1}}^{\infty} \frac{i}{n}(-1)^n e^{nix} + \frac{3}{2} i e^{-ix} - \frac{3}{2} i e^{ix}$$

$$A_n = \sqrt{a_n^2 + b_n^2} = \frac{2}{n}(-1)^n$$

$n \neq 1$

$n \neq 0$

$A_0 = a_0/2 = 0 \qquad A_1 = \sqrt{a_0^2 + 3^2} = 3$

دامنه ی

$$\Rightarrow F(x) = \sum_{n=2}^{\infty} \frac{2}{n}(-1)^n \cos\left(nx - \frac{\pi}{2}\right)$$

مرکب:

$\beta_n = tg^{-1}(\infty) = \frac{\pi}{2} \rightarrow \alpha_n = 0$

با توجه به فرد بودن تابع سری Sin ی این تابع همان سری فوریه خود تابع می شود.

سری cos ی

$F(x) = x + sainx \qquad 0 < x < 2\pi$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}(x + \sin x)dx = 2\pi$$

$$a_n = \frac{1}{\pi}\left[\int_0^{2\pi} x\cos nx dx + \int_0^{2\pi} \sin x \cos nx\right] = \frac{4}{\pi(n^2-4)n^2}\left(2n^2(-1)^n - 2n^2 + 4 - 4) - 1\right)$$

$$= \begin{cases} n = eren \Rightarrow 0 \\ n = odd = \dfrac{4(8-4n^2)}{\pi(n^2-4)n^2} \end{cases}$$

$a_n = 0 \qquad n \neq 2$

$$F(x)=\pi+\sum_{n=1}^{\infty}a_n\cos nx$$

اتحاد پارسوال

$$\frac{1}{l}\int F^2(x)dx=a_{0/2}^2+a_1^2+b_1^2+\sum_{n=2}^{\infty}\frac{4}{n^2} \qquad \sum_{n=1}^{\infty}\frac{4}{n^2}=\frac{2}{3}\pi^2$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}x^2+\sin^2 x+2x\sin x=0+0+3^2+\frac{2}{3}\pi^2-4 \qquad -4+\sum\frac{4}{n^2}=\frac{2}{3}\pi^2-4$$ 4- را از

$$\frac{1}{\pi}\left(2\pi^3/3+5\pi\right)=\frac{2}{3}\pi^2+5=9+\left(\frac{2}{3}\pi^2-4\right) \qquad \sum_{n=2}^{\infty}\frac{4}{n^2}=\frac{2}{3}\pi^2-4$$

طرفین کم می کنیم.

سوال 11

$$F(x)=1+x^2-2x \qquad -\pi<x<\pi$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(1+x^2-2x\right)dx=\frac{1}{\pi}\left(x+\frac{x^3}{3}-x\right)\Big|_{-\pi}^{\pi}=\frac{1}{\pi}\left(\frac{\pi^3}{3}+\frac{\pi^3}{3}+\pi+\pi\right)=2+\frac{2\pi^2}{3}$$

$$a_{0/2}=1+\pi^2/3$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\cos nx+x^2\cos nx-2x\cos nx\right)dx=\frac{2}{\pi}\int_0^{\pi}(\cos nx+x^2\cos n\pi)$$

فرد

$$=\frac{2}{\pi}\left\{\frac{\sin nx}{n}+\frac{x^2\sin nx}{n}+\frac{2x\cos nx}{n}-\frac{2\sin nx}{n}\right\}\Bigg|_0^{\pi}=\left(\frac{2\pi(-1)^n}{n^2}\right)\times\frac{2}{\pi}=\frac{4(-1)^n}{n^2}$$

سری فوریه: $F(x)=1+\pi^2/3+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos n+\frac{4(-1)^n}{n}\sin nx$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx + x^2 \sin nx - 2n\sin nx)dx = -\frac{4}{\pi}\int_{0}^{\pi} \sin nxdx = \frac{4(-1)^n}{n}$$

$$C_n = \frac{1}{2}\left\{\frac{4(-1)^n}{n^2} - i\,\frac{4(-1)^n}{n}\right\} = \frac{n(-1)^n}{n}\left(\frac{1}{n} - i\right)$$

$$C_0 = a_{0/2} = 1 + \frac{\pi^2}{3}$$

$$F(x) = 1 + \frac{\pi^2}{3} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{4}{n^2}\sqrt{1+n^2}\cos\left(nx - tg^{-1}\left(\frac{1}{n}\right)\right)$$ سری نمایی فوریه :

$$A_n = \sqrt{a_n^0 + b_n^2} = \sqrt{\frac{16}{n^4} + \frac{16}{n^2}} = \frac{4}{n}\sqrt{\frac{n^2}{n^2} + \frac{1}{n^2}} = \frac{4}{n^2}\sqrt{1+n^2}$$

$$A_0 = a_{0/2} = 1 + \frac{\pi^2}{3}$$

$$\beta_n = tg^{-1}\left(\frac{\frac{4(-1)^n}{n^2}}{\frac{4(-1)^n}{n}}\right) = tg^{-1}\left(\frac{1}{n}\right)$$

$$F(x) = 1 + \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}\sqrt{1+n^2}\cos\left(nx - tg^{-1}\left(\frac{1}{n}\right)\right)$$ دامنه ی

مرکب:

$x = 0 \Rightarrow F(0) = 1$ قضیه دیریکله :

پیوسته :

$$1 = 1 + \pi^2/3 + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \Rightarrow$$

$$\sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} = -\pi^2/3 \Leftrightarrow \sum_{n=1}^{\infty} (-1)^n/n^2 = -\pi^2/12$$

یعنی اتحاد $\Rightarrow 1 = 1$

درست است.

تابع جدید را می نویسیم سپس به جای $n/2, n$ b_n, a_n جایگذاری کرده و حدها را جایگذاری می کنیم.

$$a_0 = \frac{1}{\pi}\left\{\int_0^{\pi} (x-1)^2 dx + \int_{\pi}^{2\pi} (x-2\pi-1)^2 dx\right\} = \frac{1}{3\pi}\left\{(\pi-1)^3 + (\pi+1)^3\right\}$$

$$a_n = \frac{1}{\pi}\left\{\int_0^{\pi} (x-1)^2 \cos nx/2\, dx + \int_{\pi}^{2\pi} (x-2\pi-1)^2 \cos nx/2\, dx\right\} = \frac{2}{n^3\pi}\left(4n + 8n\pi\cos n\pi/2 - 4n^2\pi\sin n\pi/2 - 4n(-1)^n\right)$$

$$b_n = \frac{1}{\pi}\left\{\int_0^{\pi} (x-1)^2 \sin nx/2\, dx + \int_{\pi}^{2\pi} (x-2\pi-1)^2 \sin nx/2\, dx\right\} = \frac{2}{n^3\pi}\left(n^2 - 8 + 8n\pi\sin n\pi/2 + \right.$$

$$\left. 4n^2\pi\cos n\pi/2 + 8(-1)^n - n^2(-1)^n\right)$$

$F(x) = \sum_{n=1}^{\infty} b_n \sin nx$ سری سینوسی

$F(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos nx$ سری کسینوسی

$F(x) = x^3 \qquad 0 < x < \pi \qquad l = \pi/2$

$$a_0 = 2/\pi \int_0^{\pi} x^3 dx = 2/\pi\left(\pi^4/4\right) = \pi 3/2$$

$$a_n = 2/\pi \int_0^{\pi} x^3 \cos 2nx\, dx = 2/\pi\left\{\frac{x^3 \sin 2nx}{2n} + \frac{3x^2 \cos 2nx}{4n^2} - \frac{6x \sin 2nx}{8n^3} - \frac{6\cos 2nx}{16n^4}\right\}\Bigg|_0^{\pi}$$

$$= 2/\pi\left(3\pi^2/2n^2\right) = \frac{3\pi}{2n^2} \qquad n \neq 0$$

$$b = \frac{2}{\pi}\int_0^{\pi} x\sin 2nx\,dx = \frac{2}{\pi}\left\{\frac{-x^3\cos nx}{2n} + \frac{3x^2\sin 2nx}{4n^2} + \frac{6n\cos 2nx}{8n^3} - \frac{6\sin 2nx}{16n^4}\right\}_0^{\pi}$$

$$= \frac{2}{\pi}\left\{-\frac{\pi^3}{2n} + \frac{6\pi}{8n^3}\right\} = \frac{6}{4n^3} - \frac{\pi^2}{n}$$

$$F(x) = \frac{\pi^3}{4} + \sum_{n=1}^{\infty}\frac{3\pi}{2n^2}\cos 2nx + \left(\frac{6}{4n^3} - \frac{\pi^2}{n}\right)\sin 2nx$$: سری فوریه

$$C_n = \frac{1}{2}\left\{\frac{3\pi}{2n^2} - i\left(\frac{6}{4n^3} - \frac{\pi^2}{n}\right)\right\}$$

$$C_0 = \frac{a_0}{2} = \pi\frac{3}{4}$$

$$F(x) = \pi\frac{3}{4} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\frac{1}{2}\left\{\frac{3\pi}{2n^2} - i\left(\frac{6}{4n^3} - \frac{\pi^2}{n}\right)\right\}e^{2nix}$$: سری نمايي فوریه

$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{9\pi^2}{4n^4} + \left(\frac{6}{4n^3} + \frac{\pi^2}{n}\right)^2}$$

$$A_0 = \frac{a_0}{2} = \pi\frac{3}{4}$$

$$\beta_n = tg^{-1}\left(\frac{\frac{6}{4n^3} + \frac{\pi^2}{n}}{\frac{3\pi}{2n^2}}\right)$$

دامنه ی مرکب:

$$F(x) = \pi\frac{3}{4} + \sum_{n=1}^{\infty}\sqrt{\frac{9\pi^2}{4n^4} + \left(\frac{6}{4n^3} - \frac{\pi^2}{n}\right)^2}\cos\left\{2n\pi - tg^{-1}\left(\frac{\frac{6}{4n^3} + \frac{\pi 2}{n}}{\frac{3n}{2n^2}}\right)\right\}$$

سری cos ی :

$$a_n = \frac{2}{\pi}\left\{\frac{x^3 \sin nx}{n} + \frac{3x^2 \cos nx}{n^2} - \frac{6x \sin nx}{n^3} - \frac{6\cos nx}{n^4}\right\} = \frac{6\pi(-1)^n}{n^2}$$

$$F(x) = \pi^3\!/4 + \sum_{n=1}^{\infty} \frac{6\pi(-1)^n}{n^2}\cos nx$$

سری sin ی :

$$b_n = \frac{2}{\pi}\left\{\frac{-x^3 \cos nx}{n} + \frac{3x^2 \sin nx}{n^2} + \frac{6n\cos nx}{n^3} - \frac{6\sin nx}{n^4}\right\}_0^{\pi} = \frac{12(-1)^n}{n^3} - \frac{2\pi^2}{n}(-1)^n$$

$$F(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

اتحاد پارسوال :

$F(x) = x^3 \qquad 0 < x < \pi$

$$a_0 = \pi^3\!/4 \qquad \frac{1}{l}\int_{-e}^{e} F^2(x) = a_0^0\!/2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

$$a_n = \frac{3\pi}{2n^2} \qquad \frac{1}{\pi}\int_0^{\pi} x^6 dx = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\left(\frac{3\pi}{2n^2}\right)^2 + \left(\frac{6}{n^3} - \frac{\pi^2}{n}\right)^2\right]$$

$$b_n = \frac{6}{n^3} - \frac{\pi^2}{n} \qquad \frac{\pi}{56} = \sum_{n=1}^{\infty}\left[\left(\frac{3\pi}{2n^2}\right)^2 + \left(\frac{6}{n^3} - \frac{\pi^2}{n}\right)^2\right]$$

قسمت الف و ب- سوال 13

$F(x) = cod^2 x \qquad 0 < x < \pi \qquad 2e = \pi$

$$a_0 = \frac{2}{\pi}\int_0^\pi \cos^2 x dx = \frac{1}{\pi}\int_0^\pi (1+\cos 2x)dx = \frac{1}{\pi}\left(x + \frac{1}{2}\sin 2x\right)_0^\pi = 1$$

$$a_n = \frac{2}{\pi}\int_0^\pi \cos^2 x \cos 2nx dx = \frac{1}{\pi}\left(\int_0^\pi \cos 2nx + \int_0^\pi \cos 2x \cos 2nx\right)$$

$$a_n = \begin{cases} n \neq 1 & 0 \\ n = 1 & \frac{1}{2} \end{cases}$$

$$b_n = \frac{2}{\pi}\int_0^\pi \cos^2 x \sin 2nx dx = \frac{1}{\pi}\left(\int_0^\pi \sin 2nx + \int_0^\pi \cos 2x \sin 2nx\right) = 0$$

F.S $\Rightarrow F(x) = \frac{1}{2} + \frac{1}{2}\cos 2x$

$C_0 = \frac{1}{2}(a_0 - eb_0) = \frac{1}{2}$ $C_n = 0$ $n \neq 0,1$ برای $C_1 = \frac{1}{2}(a_1 - eb_1) = \frac{1}{4}$

F.C.E $\Rightarrow F(x) = \frac{1}{2} + \frac{1}{4}e^{2ix}$

$A_0 = \frac{a_0}{2} = \frac{1}{2}$ $A_1 = \sqrt{a_1^2 + b_1^2} = \frac{1}{2}$ $A_n = 0$ $n \neq 0,1$ برای

$$\beta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right) = \tan^{-1}(0) = 0$$

سری هارمونیک $\Rightarrow \frac{1}{2} + \frac{1}{2}\cos 2x$

برای نوشتن سری کسینوسی ، F(x) را نیمی از یک تابع زوج با دوره ی متناوب 2π فرض می کنیم داریم:

$F(x) = \cos^2 x$ $-\pi < x < \pi$ $2e = 2\pi$ $b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^\pi \cos^2 x dx = 1$$

برای محاسبه ی a_n فقط قبل از جایگذاری حدود در قسمت اول به جای n, $\frac{n}{2}$ قرار می دهیم:

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} \cos nx dx + \int_0^{\pi} \cos nx \cos 2x dx\right] \to a_n = \begin{cases} 0 & n \neq 2 \\ \frac{1}{2} & n = 2 \end{cases}$$

$$\text{F.C.S} \to F(x) = \frac{1}{2} + \frac{1}{2}\cos 2x$$

برای نوشتن سری سینوسی F(x) را نیمی از یک تابع فرد با دوره ی تناوب 2π در نظر می گیریم داریم:

$$F(x) = \begin{cases} -\cos^2 x & -\pi < x < 0 \\ \cos^2 x & 0 < x < \pi \end{cases} \qquad 2e = 2\pi \qquad a_n = 0$$

در محاسبه ی b_n فقط قبل از جایگذاری حدود در قسمت اول به جای n, $\frac{n}{2}$ قرار می دهیم:

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} \sin nx dx + \int_0^{\pi} \cos 2x \sin nx dx\right.$$

$$I = \frac{1}{\pi}\left(-\frac{1}{2}\cos nx\right)_0^{\pi} = \frac{1}{n\pi}\left(1-(-1)^n\right) = \begin{cases} \text{زوج } n \to 0 \\ \text{فرد } n \to \frac{2}{n\pi} \to \frac{2}{(2n-1)\pi} \end{cases}$$

$$II = \frac{1}{\pi}\int_0^{\pi} \sin(n+2)x - \sin(n-2)dx = \frac{1}{\pi}\left[\frac{-\cos(n+2)x}{n+2} + \frac{\cos(n-2)x}{n-2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-(-1)^{n+2}}{n+2} + \frac{1}{n+2} + \frac{(-1)^{n-2}}{n-2} - \frac{1}{n-2}\right] = \frac{1}{n+2}\left(1-(-1)^{n+2}\right) + \frac{1}{n-2}\left((-1)^{n-2} - 1\right)$$

$$\begin{cases} \text{زوج } n \to 0 \\ \\ \end{cases}$$