$$\text{فرد } n \rightarrow \frac{2}{n+2}-\frac{2}{n-2}=\frac{-8}{n^2-4}-\frac{-8}{[(2n-1)-4]\pi}$$

$$\Rightarrow b_{2n-1}=\frac{2}{(2n-1)\pi}-\frac{8}{[(2n-1)^2-4]\pi}$$

$$\text{F.S.S} \Rightarrow F(x)=\sum_{n=1}^{\infty} b_{2n-1}\sin(2n-1)x$$

بررسی همگرايي: از روی سری

$x=0 \rightarrow \frac{F(0^+)+F(0^-)}{2}=1$ میانگین حد چپ در راست $1=\frac{1}{2}+\frac{1}{2}$

همگرا $\longrightarrow$

$x=\pi \rightarrow \frac{F(\pi^+)+F(\pi^-)}{2}=1$ $\qquad 1=\frac{1}{2}+\frac{1}{2}$

$x=\frac{\pi}{2} \rightarrow F\left(\frac{\pi}{2}\right)=0$ $\qquad 0=0$

در نقاط پیوستگی به مقدار تابع در آن نقطه و در نقاط ناپیوستگی به میانگین حد چپ در راست در آن نقطه همگرا است.

قسمت الف ، ب سوال 14

$F(x)=\cos x\cos 2x \qquad 0<x<\pi \qquad e=\frac{\pi}{2}$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}\cos x\cos 2x\,dx = \frac{1}{\pi}\int_0^{\pi}(\cos x + \cos 3x)\,dx = \frac{1}{\pi}\left[\sin x + \frac{1}{3}\sin 3x\right]_0^{\pi} = 0$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi}\cos x\cos 2x\,dx + \int_0^{\pi}\cos 3x\cos 2nx\right] = 0$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi}\cos x\sin 2nx + \int_0^{\pi}\cos 3x\sin 2nx\right]$$

I II

$$I = \frac{1}{2\pi}\int_0^{\pi}(\sin(2n+1)x - \sin(2n-1)x)\,dx = \frac{1}{2\pi}\left[\frac{-\cos(2n+1)x}{2n+1} + \frac{\cos(2n-1)x}{2n-1}\right]_0^{\pi}$$

$$= \frac{2}{2n+1} - \frac{2}{2n-1} = \frac{-2}{\pi(4n^2-1)}$$

$$II = \frac{1}{2\pi}\int_0^{\pi}(\sin(2n+3)x - \sin(2n-3)x)\,dx = \frac{1}{2\pi}\left[\frac{-\cos(2n+3)x}{2n+3} + \frac{\cos(2n-3)x}{2n-3}\right]_0^{\pi}$$

$$= \frac{2}{2n+3} - \frac{2}{2n-3} = \frac{-6}{\pi(4n^2-9)} \qquad \Rightarrow b_n = \frac{-1}{\pi}\left(\frac{2}{(4n^2-1)} + \frac{6}{(4n^2-9)}\right)$$

F.S $\Rightarrow F(x) = \sum_{n=1}^{\infty} -\frac{1}{\pi}\left(\frac{2}{4n^2-1} + \frac{6}{4n^2-9}\right)\sin 2nx$,

F.S.S

$$C_n = \frac{1}{2}(a_n - eb_n) = -\frac{e}{2}b_n$$

F.C.E $\Rightarrow F(x) = \sum_{n=1}^{\infty}\frac{e}{2\pi}\left(\frac{2}{4n^2-1} + \frac{6}{4n^2-9}\right)e^{2nix}$

$$A_n = \sqrt{a_n^2 + b_n^2} = b_n \qquad \beta_n = Arc\tan\left(\frac{b_n}{a_n}\right) = \tan^{-1}(\infty) = \frac{\pi}{2} \qquad A_0 = 0$$

سری هارمونیک $\Rightarrow F(x)=\sum_{n=1}^{\infty}-\frac{1}{\pi}\left(\frac{2}{4n^2-1}+\frac{6}{4n^2-9}\right)\cos\left(2nx-\frac{\pi}{2}\right)$

اتحاد پارسوال:

$$\frac{2}{\pi}\int_0^{\pi}\cos^2 x\cos^2 2x=\sum_{n=1}^{\infty}b_n^2$$

I

$$I=\frac{1}{4\pi}\left[\int_0^{\pi}(1+\cos 2x)(1+\cos 4x)dx=\frac{1}{4\pi}\left[\int_0^{\pi}(1+\cos 2x+\cos 4x+\cos 2x\cos 4x)dx\right.\right.$$

$$=\frac{1}{4\pi}\left[x+\frac{1}{2}\sin 2x+\frac{1}{4}\sin 4x\right]_0^{\pi}=\frac{1}{4}$$

$$\frac{1}{4}=\frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{2}{4n^2-1}+\frac{6}{4n^2-3}\right)$$

برای نوشتن سری کسینوسی کافی است در a_n قسمت اول

به جای n، قبل از جایگذاری حدود $\frac{n}{2}$ قرار دهیم:

$F(x)=\cos x.\cos 2x \qquad -\pi<x<\pi \qquad b_n=0$

$$a_0=\frac{l}{\pi}\int_0^{\pi}\cos x\cos 2xdx=0$$

$a_n=\frac{1}{\pi}\left[\int_0^{\pi}\cos x\cos nx+\int_0^{\pi}\cos 3x\cos nx\right]=0$ برای $n\neq 3, n\neq 1$

$$a_1=\frac{1}{\pi}\left[\int_0^{\pi}\cos^2 x+\int_0^{\pi}\cos 3x\cos x\right]=\frac{1}{2\pi}\left(x+\frac{1}{2}\sin 2x\right)_0^{\pi}=\frac{1}{2}$$

$$a_3=\frac{1}{\pi}\left[\int_0^{\pi}\cos x\cos 3x+\int_0^{\pi}\cos^2 3x\right]=\frac{1}{2\pi}\left(x+\frac{1}{6}\sin 6x\right)_0^{\pi}=\frac{1}{2}$$

F.C.S $\Rightarrow F(x) = \frac{1}{2}\cos x + \frac{1}{2}\cos 3x$

قسمت الف و ب سوال 15

$$F(x) = x|x| \qquad -\pi < x < \pi \qquad e = \pi$$

$$F(x) = \begin{cases} -x^2 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -x^2 dx + \int_{0}^{\pi} x^2 dx\right] = \frac{1}{\pi}\left[-\frac{x^3}{3}\Big|_{-\pi}^{0} + \frac{x^3}{3}\Big|_{0}^{\pi}\right] = 0$$

I II

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -x^2 \cos nx + \int_{0}^{\pi} x^2 \cos nx\right]$$

$$I = \frac{1}{\pi}\left(-\frac{x^2}{n}\sin nx - \frac{2x}{n^2}\cos nx + \frac{2}{n^3}\sin nx\right) = \left(-\frac{2\pi}{n^2}(-1)^n\right)$$

$$II = \frac{1}{\pi}\left(\frac{x^2}{n}\sin nx + \frac{2x}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right) = \left(\frac{2\pi}{n^2}(-1)^n\right)\frac{1}{\pi}$$

$$\Rightarrow a_n = 0$$

I II

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -x^2 \sin nx + \int_{0}^{\pi} x^2 \sin nx\right]$$

$$I=\frac{1}{\pi}\left(\frac{x^2}{n}\cos nx-\frac{2x}{n^2}\sin nx-\frac{2}{n^3}\cos nx\right)_{-\pi}^{0}=\frac{1}{\pi}\left(\frac{-\pi^2}{n}(-1)^n+\frac{2}{n^3}(-1)^n-\frac{2}{n^3}\right)$$

$$II=\frac{1}{\pi}\left(\frac{-x^2}{n}\cos nx+\frac{2x}{n^2}\sin nx+\frac{2}{n^3}\cos nx\right)_{0}^{\pi}=\frac{1}{\pi}\left(\frac{-\pi^2}{n}(-1)^n+\frac{2}{n^3}(-1)^n-\frac{2}{n^3}\right)$$

$$b_n=\left(\frac{2\pi}{n}(-1)^n+\frac{4}{n^3\pi}(-1)^3-\frac{4}{n^3\pi}\right)=\left(\frac{4}{n^3\pi}-\frac{2\pi}{n}\right)(-1)^n-\frac{4}{n^3\pi}$$

سری فوریه و سری سینوسی فوریه $F(x)=\sum_{n=1}^{\infty}\left[\left(\frac{4}{n^3\pi}-\frac{2\pi}{n}\right)(-1)^n-\frac{4}{n^3\pi}\right]\sin nx$

$$C_n=\frac{1}{2}(a_n-eb_n)=-\frac{e}{2}b_n$$

سری مختلط نمایی $F(x)=\sum_{n=-\infty}^{\infty}\left[\left(\frac{-2e}{n^3\pi}+\frac{i\pi}{n}\right)(-1)^n+\frac{2i}{n^3\pi}\right]e^{enx}$

$$A=\sqrt{a_n^2+b_n^2}=b_n \qquad \tan^{-1}\left(\frac{b_n}{a_n}\right)=\tan^{-1}(\infty)=\frac{\pi}{2}=\beta_n$$

سری هارمونیک $F(x)=\sum_{n=1}^{\infty}\left[\left(\frac{4}{n^3\pi}-\frac{2\pi}{n}\right)(-1)^n-\frac{4}{n^3\pi}\right]\cos\left(nx-\frac{\pi}{2}\right)$

سری کسینوس فوریه:

$$F(x)=\begin{cases}-(x-2\pi)^2 & \pi<x<2\pi\\ x^2 & 0<x<\pi\end{cases}$$

ضابطه را نیمی از تابع زوج با دوره ی تناوب 4π در نظر می گیریم و پس از نوشتن a_n برای تابع فوق به جای n، $\frac{n}{2}$ قرار می دهیم . داریم:

$$a=\frac{1}{\pi}\left[\int_0^{\pi} x dx-\int_{\pi}^{2\pi}(x-2\pi)dx\right]=\frac{1}{\pi}\left(\frac{\pi}{3}-\frac{\pi}{3}\right)=0$$

$$a=\frac{1}{\pi}\left[\int_0^{\pi} x^2 Cos\frac{n}{2}x-\int_{\pi}^{2\pi}(x-2\pi)^2 Cos\frac{n}{2}x\right]$$

I II

$$I=\frac{1}{\pi}\left[\frac{2x^2}{n}Sin\frac{n}{2}x+\frac{8x}{n^2}Cos\frac{n}{2}x-\frac{16}{n^3}Sin\frac{n}{2}x\right]_0^{\pi}$$

$$=\frac{1}{\pi}\left[\frac{2\pi^2}{(2n-1)}(-1)^{n+1}+\frac{8\pi}{(2n)^2}(-1)^n-\frac{16}{(2n-1)^3}(-1)^{n+1}\right]$$

$$II=-\frac{1}{\pi}\left[\frac{2(x-2\pi)^2}{n}Sin\frac{n}{2}x+\frac{8(x-2\pi)}{n^2}Cos\frac{n}{2}x-\frac{16}{n^3}Sin\frac{n}{2}x\right]_{\pi}^{2\pi}$$

$$=\frac{1}{\pi}\left[\frac{2\pi^2}{(2n-1)}(-1)^{n+1}-\frac{8\pi}{(2n)^2}(-1)^n-\frac{16}{(2n-1)^3}(-1)^{n+1}\right]$$

$$a_n=\left[\frac{4\pi}{2n-1}-\frac{32}{\pi(2n-1)^3}\right](-1)^{n+1}$$

$$F(x)=\sum_{n=1}^{\infty}a_n Sin\frac{(2n-1)}{2}x$$ میانگین حد چپ و راست

$$x=0\rightarrow\frac{F(0^+)+F(0^-)}{2}=0$$ $0=0$ بررسی

همگرایی

از طریق سری

$F(x)=2x+1$ $-1<x<1$ $e=1$

$$a_0 = 2\int_{-1}^{1} x dx + \int_{-1}^{1} dx = 2$$

$$a_n = 2\int_{-1}^{1} x Cos n\pi x dx + \int_{-1}^{1} Cos n\pi x dx = 0$$

$$b_n = 2\int_{-1}^{1} x Sin n\pi x dx + \int_{-1}^{1} Sin n\pi x dx = 4\int_{0}^{1} x Sin n\pi x$$

$$= 4\left(-\frac{x}{n\pi} Cos n\pi x + \frac{1}{n^2\pi^2} Sin n\pi x\right)_0^1 = \frac{4(-1)^{n+1}}{n\pi}$$

$$\text{F.S} \Rightarrow F(x) = 1 + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n\pi} Sin n\pi x$$

$$C_n = \frac{1}{2}(a_n - eb_n) = \frac{1}{2}(0 - \frac{4i}{n\pi}(-1)^{n+1}) = \frac{2}{n\pi i}(-1)^{n+1}$$

$$\text{F.C.E} \Rightarrow F(x) = \frac{2}{i\pi}\sum_{n=-\infty}^{\infty} \frac{(-1)^{n+1}}{n} e^{ni\pi x}$$

$$A_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}} = \sqrt{\frac{16}{n^2\pi^2}} = \frac{4}{n\pi} \qquad A_0 = 1 \qquad \beta_n = \tan^{-1}(\frac{b_n}{a_n}) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

سری هارمونیک $\Rightarrow F(x) = 1 + \sum_{n=1}^{\infty} \frac{4}{n\pi} Cos(n\pi x - \frac{\pi}{2})$

اتحاد پارسوال :

$$\int_{-1}^{1} (2x-1)^2 dx = 2 + \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2}$$

$$I=\int_{-1}^{1}(4x^2+4x+1)dx=\frac{4}{3}x^3+2x^2+x\Big|_{-1}^{1}=\frac{8}{3}$$

$$\frac{8}{3}=\sum_{n=1}^{\infty}\frac{16}{n^2\pi^2}\rightarrow\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$

$$F(x)=\begin{cases}2x+1 & 0<x<1\\ 2x-3 & 1<x<2\end{cases}$$

برای نوشتن سری کسینوسی ضابطه را نیمی از تابع زوج با دوره ی تناوب 4 در نظر می گیریم و پس از نوشتن a_n برای تابع فوق به جای n، $\frac{n}{2}$ قرار دهیم داریم:

$$a_0=\int_0^1(2x+1)dx+\int_1^2(2x-3)dx=2 \qquad b_n=0$$

$$a_n=\int_0^1(2x+1)\cos\frac{n\pi}{2}xdx+\int_1^2(2x-3)\cos\frac{n\pi}{2}xdx$$

$$=2\int_0^2 x\cos\frac{n\pi}{2}xdx+\int_0^1\cos\frac{n\pi}{2}x-3\int_1^2\cos\frac{n\pi}{2}x$$

I II III

$$I=2\left[\frac{2x}{n\pi}\sin\frac{n\pi}{2}x+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}x\right]=\frac{8}{n^2\pi^2}\left[(-1)^n-1\right]$$

$$II=\frac{2}{n\pi}\sin\frac{n\pi}{2}x\Big|_0^1=\frac{2}{(2n-1)\pi}(-1)^{n+1}$$

$$III=\frac{-6}{n\pi}\sin\frac{n\pi}{2}x\Big|_1^2=\frac{6}{(2n-1)\pi}(-1)^{n+1}$$

$$F(x)=1+\sum_{n=1}^{\infty}\left[\frac{8}{n^2\pi^2}\left[(-1)^n-1\right]+\frac{8}{(2n-1)\pi}(-1)^{n+1}\right]\cos\frac{n\pi}{2}x$$ سری کسینوسی

برای نوشتن سری کسینوسی ضابطه را نیمی از تابع فرد با دوره ی تناوب 4 در نظر می گیریم و پس از نوشتن b_n برای تابع فوق به جای n, $\frac{n}{2}$ قرار می دهیم داریم:

$$b_n = 2\int_0^2 x\sin\frac{n\pi}{2}x\,dx + \int_0^1 \sin\frac{n\pi}{2}x\,dx - 3\int_1^2 \cos\frac{n\pi}{2}x\,dx$$

I II III

$$I = 2\left(-\frac{2x}{n\pi}\cos\frac{n\pi}{2}x + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}x\right)_0^2 = \frac{8}{n\pi}(-1)^{n+1}$$

$$II = -\frac{2}{n\pi}\cos\frac{n\pi}{2}x\Big|_0^1 = \frac{-2}{2n\pi}(-1)^n + \frac{2}{n\pi} = \frac{1}{n\pi}\left((-1)^{n+1}+2\right)$$

$$III = \frac{6}{n\pi}\cos\frac{n\pi}{2}x\Big|_1^2 = \frac{6(-1)^n}{n\pi} - \frac{3}{n\pi}(-1)^n = \frac{3(-1)^n}{n\pi}$$

سری سینوسی

$$F(x) = \sum_{n=1}^{\infty}\left(\frac{9}{n\pi}(-1)^{n+1} + \frac{2}{n\pi} + \frac{3(-1)^n}{n\pi}\right)\sin\frac{n\pi}{2}x$$

جواب سوال 17

$$F(x) = l_n(1+x) \qquad 0<x<1 \Rightarrow F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{l}x + b_n\sin\frac{n\pi}{l}x\right)$$

$$2l = 1 \to l = 1/2$$

$$a_0 = 2\int_0^1 l(1+x)dx = (x+1)l(x+1) - x + 1$$

$$2(x+1)\ln(x+1)-(x+1)\Big|_0^1 = \big((2\ln 2-2)-(-1)\big)=0$$

$$2(2l_n 2-1) = -2+4l_n 2 = a_0$$

$$a_n = \frac{1}{l}\int_{0+c}^{2R+c} F(x)\cos\frac{n\pi}{l}x\,dx = 2\int_0^1 l_n(1+x)\cos 2nRx\,dx$$

جواب سوال :18

$0<x<1 \qquad F(x)=\sinh(2x)=\frac{1}{2}(e^{2x}-e^{-2x})$ تابع فرد

$$2l=1\rightarrow l=\frac{1}{2}$$

$$a=\frac{1}{l}\int_c^{c+21}F(x)dx = 2\times\frac{1}{2}\int_0^1 e^{2x}-e^{-2x}$$

$$=\frac{1}{2}e^{2x}+\frac{1}{2}e^{-2x}=\frac{1}{2}\left(e^2+e^{-2}\right)-1=a_0$$

$$a_n=\frac{1}{c}\int_c^{c+21}F(x)\cos\frac{n\pi}{l}x\,dx = 2\int_0^1\frac{1}{2}(e^{2x}-e^{-2x})\cos 2n\pi x\,dx$$

$$=\int_0^1 e^{2x}\cos 2n\pi x\,dx+\int_0^1 -e^{-2x}\cos 2n\pi x\,dx$$

$I_1 \qquad\qquad I_2$

$$I_1=\left[\frac{e^{2x}}{2n\pi}\sin 2n\pi x+\frac{e^{2x}}{2n^2\pi^2}\cos 2n\pi x-\frac{1}{n^2\pi^2}I_1\right]_0^1$$

$$\left(1+\frac{1}{n^2\pi^2}\right)I_1=\left(\frac{e^2}{2n^2\pi^2}\right)-\frac{1}{2n^2\pi^2}=\frac{e^2-1}{2n^2\pi^2}$$

$$I_1=\frac{e^2-1}{2(1+n^2\pi^2)}$$

I_2

$$I_2=\frac{-e^{-2x}}{2n\pi}\sin 2n\pi x+\frac{e^{-2x}}{2n^2\pi^2}\cos 2n\pi x-\frac{1}{n^2\pi^2}\int e^{-2x}\cos 2n\pi x$$

$$I_2=\frac{e^{-2}-1}{2(1+n^2\pi^2)} \qquad\qquad I_1+I_2=\frac{1}{2(1+n^2\pi^2)}(e^2+e^{-2}-2)$$
