ابتدا تابع فرد تابع اصلی را بدست می آوریم:

$$\begin{cases} x^4-2x^3+x^2+x : 0<x<1 \\ \frac{1}{\Pi}\varepsilon.\pi(x-2) : 1<x<2 \end{cases}$$

سری فوریه: $2l=2\to l=1$

ولی باید قسمت چپ را نیز در نظر بگیریم:

$$a_0=\frac{-2}{\pi^2}+\frac{8}{15}$$

سری $\varepsilon.$ ای فوریه $2l=4\to l=2$

چون تابع فرد است:

$$a_0=\int_0^1 F(x)\cos\frac{n\pi}{l}xdx=0$$

$$b_n=\int_0^1\left(x^4-2x^3+x^2+x\right)\varepsilon.\frac{n\pi}{2}xdx+\int_1^2\varepsilon.(\pi x-2n)\varepsilon.\frac{n\pi}{2}xdx$$

I_1 $\qquad$ I_2

$$I_1=\left(\frac{-2\left(x^4-2x^3+x^2+x\right)}{n\pi}\cos\frac{n\pi}{2}x+\frac{4\left(4x^3-6x^2+2x+1\right)}{n\pi}\varepsilon.\frac{n\pi}{2}x+\frac{8\left(12n^e-12x+2\right)}{n^3\pi^3}\cos\frac{n\pi}{2}x\right.$$
$$\left.-\frac{16(24x-12)}{n^4\pi^4}\varepsilon.\frac{n\pi}{2}x-\frac{768}{n^5\pi^5}\cos\frac{n\pi}{2}x\right)_0^1=\left[-\frac{2}{n\pi}\cos\frac{n\pi}{2}+\frac{4}{n^2\pi^2}\varepsilon.\frac{n\pi}{2}+\frac{16}{n^3\pi^3}\cos\frac{n\pi}{2}\right.$$
$$\left.-\frac{192}{n^4\pi^4}\varepsilon.\frac{n\pi}{2}-\frac{768}{n^5\pi^5}\cos\frac{n\pi}{2}\right]=-\frac{16}{n^3\pi^3}+\frac{768}{n^5\pi^5}$$

$$I_2=\int_1^2 \tfrac{1}{\pi}\,\varepsilon.(\pi x-2\pi)\varepsilon.\frac{n\pi}{2}x\,dx=\int_1^2 \tfrac{1}{2\pi}\left(\cos\left(\pi x-2\pi-\frac{n\pi}{2}x\right)-\cos\left(\pi x-2\pi+\frac{n\pi}{2}x\right)\right)$$

$$=\int_1^2 \tfrac{1}{2\pi}\left[\cos x\left(\pi-\frac{n\pi}{2}\right)-\cos x\left(\pi+\frac{n\pi}{2}\right)\right]$$

$$=\tfrac{1}{2\pi}\left[\frac{1}{x\left(\pi-\frac{n\pi}{2}\right)}\varepsilon.\left(\pi x-\frac{n\pi}{2}x\right)-\frac{\varepsilon.\left(\pi x+\frac{n\pi}{2}x\right)}{\pi x+\frac{n\pi}{2}x}\right]_1^2$$

$$=-\tfrac{1}{2\pi}\left[\frac{\varepsilon.\left(\pi-\tfrac{n\pi}{2}\right)}{\pi-\tfrac{n\pi}{2}}-\frac{\varepsilon.\left(\pi+\tfrac{n\pi}{2}\right)}{\left(\frac{n\,\Pi}{2}+\pi\right)}\right]$$

$$=-\tfrac{1}{2\Pi}\left(\frac{\varepsilon.n\tfrac{\pi}{2}}{n-\tfrac{n\pi}{2}}+\frac{\varepsilon.\tfrac{n\pi}{2}}{n+\tfrac{n\pi}{2}}\right)=\frac{\varepsilon.\tfrac{n\pi}{2}}{\frac{n^2\pi^2}{4}-\pi^2}$$

$$b_n=I_1+I_2 \qquad F(x)=\sum_{n=1}^{\infty} b_n\varepsilon\frac{n\pi}{2}x$$

سری cos ای فوریه :

$$b_n=0$$

$$a_n=\int_0^1\left(x^4-2x^3+x^2+x\right)\cos\frac{n\pi}{2}x\,dx+\int_1^2 \tfrac{1}{\pi}\varepsilon.(\pi x-2\pi)\cos\frac{n\pi}{2}x\,dx$$

I_1 $\qquad\qquad$ I_2

$$I_1=\left[\frac{2\left(x^4-2x^3+x^2+x\right)}{n\pi}\varepsilon.\frac{n\pi}{2}x+\frac{4\left(4x^3-6x^2+2x+1\right)}{n^2\pi^2}\cos\frac{n\pi}{2}x-8\frac{\left(12x^2-12x+4\right)}{n^3\pi^3}\varepsilon.\frac{n\pi}{2}x\right.$$

$$\left.-\frac{16(24x-12)}{n^4\pi^4}\cos\frac{n\pi}{2}x+\frac{768}{n^5\pi^5}\varepsilon.\frac{n\pi}{2}x\right.$$

$$=\left[\frac{2}{n\pi}\varepsilon.\frac{n\pi}{2}+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}-\frac{16}{n^3\pi^3}\varepsilon.\frac{n\pi}{2}-\frac{19}{n^4\pi^4}\cos\frac{n\pi}{2}+\frac{768}{n^5\pi^5}\varepsilon.\frac{n\pi}{2}\right]_0^1$$

$$-\left[\frac{4}{n^2\pi^2}+\frac{192}{n^4\pi^4}\right]$$

$$I_2 = \int_1^2 \tfrac{1}{\pi}\, \varepsilon.(\pi x - 2\pi)\cos \frac{n\pi}{2} x = \int_1^2 \tfrac{1}{\pi}\, \varepsilon\, \pi x \cos \frac{n\pi}{2} x$$

$$= \int_1^2 \tfrac{1}{2\pi} \left[\varepsilon.\left(\pi x + \frac{n\pi}{2} x \right) + \varepsilon.\left(\pi x - \frac{n\pi}{2} x \right) \right]_1^2$$

$$= \tfrac{1}{2\pi} \left[\frac{-1}{\pi + \frac{n\pi}{2}} \cos\left(\pi x + \frac{n\pi}{2} x \right) - \frac{1}{\pi - n\pi/2} \cos\left(\pi x - \frac{n\pi}{2} x \right) \right]_1^2 =$$

$$\tfrac{1}{2\pi} \left[(-1)^{n+1} \left(\frac{1}{\pi + n\pi/2} + \frac{1}{\pi - n\pi/2} \right) + \frac{-\cos n\pi/2}{\pi + n\pi/2} + \frac{\cos \pi/2}{n - n\pi/2} \right]$$

$$\tfrac{1}{2\pi} \left[\frac{-1}{\pi + n\pi/2} \cos\left(\pi x + \frac{n\pi}{2} x \right) - \frac{1}{\pi - n\pi/2} \cos\left(\pi_x - \frac{n\pi}{2} x \right) \right]$$

$$= \tfrac{1}{2} \left[\frac{1}{\pi + n\pi/2} + \frac{1}{\pi - n\pi/2} \right] \left[(-1)^{n+1} \cos \frac{n\pi}{2} \right]$$

$$= \frac{(-1)^{n+1} - \cos n\pi/2}{n^2 - n^2 \pi^2/4}$$

$$a_n = I_1 + I_2$$

$$F(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} x dx$$

سرى مختلط نمايي فوريه :

$$C_n = \tfrac{1}{2}(a_n - ib_n) = \begin{cases} \tfrac{1}{2}\left[\dfrac{-24}{n^4\pi^4} - \dfrac{2}{\pi^2 - n^2\pi^2} + \dfrac{i}{n\pi} \right] & n : even \\ \tfrac{1}{2}\left[\dfrac{-2}{n^2\pi^2} - i\left(\dfrac{-4}{n^3\pi^3} + \dfrac{48}{n^5\pi^5} + \dfrac{1}{n\pi} \right) \right] & n : odd \end{cases} \qquad n \neq 0$$

$$C_0 = \frac{a_0}{2} = \frac{4}{15} - \frac{1}{\pi^2}$$

$$F(x) = C_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} C_n e^{n\Pi/l\, in}$$

Compound Amplitude

$$A_n = \sqrt{a_n^2 + b_n^2} \qquad F(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{l}x - \beta_x\right)$$

$$\beta_n = tg^{-1}\left(b_n \big/ a_n\right)$$

پارسوال

$$a_0 = \frac{8}{15} - 2\big/\Pi^2 \qquad a_n = \begin{cases} \dfrac{-24}{n^4\Pi^4} - \dfrac{2}{\Pi^2 - \Pi^2 n^2} : n = even \\ \dfrac{-2}{n^2\Pi^2} n = odd \end{cases}$$

$$b_n = 0 \qquad 1\big/l \int_{-l}^{l} F^2(x)dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

$$F^2(x) = \begin{cases} 1\big/\pi \, \varepsilon.\pi x & -1 < x < 0 \\ x^4 - 2x^3 + x^2 + x & 0 < x < 1 \end{cases}$$

$$F^2(x) = \begin{cases} 1\big/\pi^2 \, \varepsilon.^2 \pi n \\ x^8 - 4x^7 + 6x^6 - 2x^5 - 3x^4 + 2x^3 + x^2 \end{cases} \qquad -1 < x < 1$$

$$1\big/l \int_{-l}^{l} F^2(x)dx = \left[\int_{-1}^{0} 1\big/\pi^2 \, \varepsilon.^2 \pi x dx + \int_{0}^{1} I dx\right]$$

طرف اول: $I_1 = 1\big/l \int_{-1}^{0} 1 - \cos 2\pi x = 1\big/2\pi^2 \left(x - \frac{1}{2\pi}\varepsilon.2\pi x\right)_{-1}^{0} = -\frac{1}{2\pi^2}(-1) = 1\big/2\pi^2$

$$I_2 = \left(\frac{x^9}{9} - 1\big/2\, x^8 + 6\big/7\, x^7 - 1\big/3\, x^6 - 3\big/5\, x^5 + 1\big/2\, x^4 + x^3\big/3\right)_0^1 = \frac{116}{315}$$

طرف اول $\rightarrow I_1 + I_2 = \frac{1}{2\pi^2} + \frac{116}{315}$

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21) $2l = 2\pi \qquad -\pi < x < 0 \qquad f(x) = \begin{cases} 1 + x^2 \\ x \end{cases}$

$l = \pi \qquad 0 < x < \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n Cosnx + b_n Sinnx) \quad \text{F.S. :}$$

$$a_0 = \frac{1}{\pi}\left\{ \int_{-\pi}^{0} (1 + x^2)dx + \int_{0}^{\pi} xdx \right\} = \frac{1}{\pi}\left\{ x + \frac{x^3}{3} \Big]_{-\pi}^{0} + \frac{x^2}{2} \Big]_{0}^{\pi} \right\}$$

$$= \frac{1}{\pi}\left\{ \pi + \frac{\pi^3}{3} + \frac{\pi^2}{2} \right\} = 1 + \frac{\pi^2}{3} + \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\left\{ \int_{-\pi}^{0} (1 + x^2)Cosnxdx + \int_{0}^{\pi} Cosnxdx \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{Sinnx}{n} + \frac{x^2}{n} Sinnx + \frac{2x}{n^2} Cosnx - \frac{2}{n^3} Sinnx \Big]_{-\pi}^{0} + \frac{x}{n} Sinnx + \frac{1}{n^2} Cosnx \Big]_{0}^{\pi} \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{2\pi}{n^2}(-1)^n + \frac{1}{n^2}(-1) - \frac{1}{n^2} \right\} = \begin{cases} \dfrac{2}{n^2} & n = even \\ \dfrac{-2(\pi + 1)}{\pi n^2} & n = odd \end{cases}$$

$$b_n = \frac{1}{\pi}\left\{ \int_{-\pi}^{0} (1 + x^2)Sinnxdx + \int_{0}^{\pi} xSinnxdx \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{-\cos nx}{n} - \frac{x^2}{x}\cos nx + \frac{2x}{n^2}\sin nx + \frac{2}{n^3}\cos nx \Big]_{-\pi}^{0} - \frac{x}{n}\cos nx + \frac{\sin nx}{n^2} \Big]_{0}^{\pi} \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{-1}{n} + \frac{(-1)^n}{n} + \frac{\pi^2}{n}(-1)^{-1} + \frac{2(-1)^{n+1}}{n^3} + \frac{2}{n^3} - \frac{\pi}{n}(-1)^n \right\}$$

$$= \begin{cases} \dfrac{\pi - 1}{n} & n = even \\ \dfrac{-(\pi^2 + 2 - \pi)}{n\pi} + \dfrac{4}{n^3\pi} & n = odd \end{cases}$$

F.C.S.:

$$f(x)=\begin{cases}(x-\pi)+1 & 0<x<\pi\\ x-\pi & \pi<x<2\pi\end{cases}$$

$2l=4\pi$

$l=2\pi$

$$a_0=\frac{1}{\pi}\left\{\int_0^{\pi}(x-\pi)^2+1dx+\int_{\pi}^{2\pi}(x-\pi)dx\right\}$$

$$=\frac{1}{\pi}\left\{\frac{(x-\pi)^2}{2}+x\Big]_0^{\pi}+\frac{x^2 2}{}-\pi x\Big]_{\pi}^{2\pi}\right\}=\frac{1}{\pi}\left\{\pi-\frac{\pi^2}{2}+\frac{4\pi^2}{2}-2\pi^2-\frac{\pi^2}{2}+\pi^2\right\}=1$$

$$a_n=\frac{1}{\pi}\left\{\int_0^{\pi}\left((x-\pi)^2+1\right)\cos{}^{n}\!/_{2}\,xdx+\int_{\pi}^{2\pi}(x-\pi)\cos{}^{n}\!/_{2}\,xdx\right\}$$

$$=\frac{1}{\pi}\left\{\frac{2(x-\pi)^2}{n}\sin{}^{n}\!/_{2}\,x+\frac{8(x-\pi)}{n^2}\cos{}^{n}\!/_{2}-\frac{16}{n^3}\sin{}^{n}\!/_{2}\,x+\frac{2}{n}\sin{}^{n}\!/_{2}\,x\Big]_0^{\pi}+\frac{2x}{n}\sin{}^{n}\!/_{2}\,x\right.$$

$$\left.+\frac{4}{n}\cos{}^{n}\!/_{2}\,x-\frac{2\pi}{n}\sin{}^{n}\!/_{2}\,x\Big]_{\pi}^{2\pi}\right\}=$$

$$=\frac{1}{\pi}\left\{\frac{-8\pi}{n^2}-\frac{16}{n^3}\sin{}^{n}\!/_{2}\,\pi+\frac{2}{n}\sin{}^{n}\!/_{2}-\frac{2\pi}{n}\sin{}^{n}\!/_{2}\,\pi+\frac{4}{n}(-1)^n-\frac{4}{n}\cos{}^{n}\!/_{2}\,\pi+\frac{2\pi}{n}\sin{}^{n}\!/_{2}\,\pi\right\}$$

$$=\begin{cases}\frac{1}{\pi}\left(\frac{-8\pi}{n^2}+\frac{4}{n^2}\right)\left(1-(-1)^2\right) & n=even\\ \frac{1}{\pi}\left(\frac{-(8\pi+4)}{n^2}+(-1)^n\left(\frac{2}{n}-\frac{16}{n^3}\right)\right) & n=odd\end{cases}$$

F.S.S.:

$$b=\frac{1}{\pi}\left\{\int_0^{\pi}\left((x-\pi)^2+1\right)\sin{}^{n}\!/_{2}\,xdx+\int_{\pi}^{2\pi}(x-\pi)\sin{}^{n}\!/_{2}\,xdx\right\}$$

$$= \frac{1}{\pi}\left\{\frac{-2(x-\pi)^2}{n}\cos {n}/{2}\,x + \frac{8(x-\pi)}{n^2}\sin {n}/{2} + \frac{16}{n^3}\cos {n}/{2}\,x - \frac{2}{n}\cos {n}/{2}\,x \;\Big]_0^{\pi} - \frac{2x}{n}\cos {n}/{2}\,x\right.$$

$$\left. + \frac{4}{n^2}\sin {n}/{2}\,x + \frac{2\pi}{n}\cos {n}/{2}\,x \;\Big]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{-2\pi^2}{x} + \frac{16}{n^3}\cos {n}/{2}\,\pi - \frac{16}{n^3} - \frac{2}{n}\cos\frac{n}{2}\pi + \frac{2}{n} - \frac{4\pi}{n}(-1)^n + \frac{2\pi}{n}\cos {n}/{2}\,\pi - \frac{4}{n^2}\sin {n}/{2}\,\pi\right.$$

$$\left. + \frac{2\pi}{n}(-1)^n - \frac{2\pi}{n}\cos {n}/{2}\,\pi\right\} = \begin{cases} \frac{1}{\pi}\left(\frac{-2\pi(\pi+1)}{n} + \frac{16}{n^3}\left((-1)^n - 1\right) + \frac{2}{n}\left(1-(-1)\right)\right) & n = even \\ \frac{1}{\pi}\left(\frac{-2(\pi^2-\pi-1)}{n} - \frac{4}{n^2}(-1)^n - \frac{16}{n^3}\right) & n = odd \end{cases}$$

F.C.E. $F(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{ni\pi}{e}x}$

$$C_n = \frac{1}{2\pi}\left\{\int_0^{\pi}\left((x-\pi)^2+1\right)e^{-nix}dx + \int_{\pi}^{2\pi} xe^{-nix}dx\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{(x-\pi)^2}{ni}e^{-nix} + \frac{2(x-\pi)}{n^2}e^{-nix} + \frac{2e^{-nix}}{n^3 i} - \frac{e^{-nix}}{ni}\;\Big]_0^{\pi} - \frac{xe^{-nix}}{ni} + \frac{e^{-nix}}{n^2}\;\Big]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{\pi^2}{ni} - \frac{\pi}{n^2} + \frac{2(-1)^n}{n^3 i} - \frac{2}{n^3 i} - \frac{((-1)^n - 1)}{ni} - \frac{2\pi - \pi(-1)^n}{ni} + \frac{1-(-1)^n}{n^2}\right\}$$

Harmonic: $F(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{l}x - \beta_n\right)$

$$A_0 = \frac{a_0}{2} = \frac{1}{2}\left(1 + {\pi^2}/{3} + {\pi}/{1}\right)$$

$$A_n^2 = a_n^2 - b_n^2 \rightarrow A_n = \begin{cases} \sqrt{\frac{4}{n^4} + \frac{(\pi+1)^2}{n^2}} & n = even \\ \sqrt{\frac{4(\pi+1)^2}{\pi^2 n^4} + \frac{n^4(\pi+2-\pi)^2 + 16}{n^6\pi^2}} & n = odd \end{cases}$$

$$\beta_n = tg^{-1}\left({b_n}/{a_n}\right)$$

ب) $F(x)=1+\pi^2/3+\pi/2+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

$x=0$ $\qquad$ $\frac{1}{2}=1+\pi^2/3+\pi/2+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

نقطه ناپیوسته

$x=\pi/2$ $\qquad$ $\pi/2=1+\pi^2/3+\pi/2+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

$$-1-\pi^2/2=\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

$$\frac{2}{e}\left[\frac{xl}{n\pi}\sin\frac{n\pi}{e}x+\frac{ae^2}{n^2\pi^2}\cos\frac{n\pi}{e}x\right]_0^{e/2}+\frac{ae^2}{n\pi}\sin\frac{n\pi}{e}x-\frac{axl}{n\pi}\sin\frac{n\pi}{e}x-\frac{al^2}{n^2\pi^2}\cos\frac{n\pi}{e}x\Big]_{e/2}^{e}$$

$$=\frac{2}{e}\left\{\frac{ae^2}{2n\pi}\sin n\pi/2+ae^2/n^2\pi^2\cos n\pi/2-\frac{ae^2}{n^2\pi^2}-\frac{ae^2}{n\pi}\sin n\pi/2+\frac{ae^2}{2n\pi}\sin\frac{n\pi}{2}-\frac{ae^2}{n^2\pi^2}(-1)^n\right.$$

$$\left.+\frac{ae^2}{n^2\pi^2}\cos n\pi/2\right\}=\frac{2}{e}\left\{\frac{2ae^2}{n^2\pi^2}\cos n\pi/2-\frac{ae^2}{n^2\pi^2}\left[1-(-1)^n\right]\right\}=$$

$$=\begin{cases}\frac{2}{e}\left(\frac{-2ae^2}{n^2\pi^2}(-_{\downarrow\cos\frac{n\pi}{2}}1)^n+\frac{ae^2}{n^2\pi^2}\right) & n=evem\\ 0 & n=odd\end{cases}$$

$$a=\begin{cases}0 & n=odd\\ 0 & n=4K\\ \frac{-8ae}{n^2\pi^2} & n=4K-2\end{cases}\quad (K=1,2,3,\ldots)$$

$$\Rightarrow F(x)=\frac{ae}{4}+\sum_{n=1}^{\infty}\frac{-8ae}{\pi^2(4n-2)^2}\cos\frac{(4n-2)\pi}{e}x$$

F.S.S

$$b_n = \frac{2}{e}\left\{\int_0^{e/2} ax\sin\frac{n\pi}{l}xdx + \int_{e/2}^{e} a(e-x)\sin\frac{n\pi}{l}xdx\right\}$$

$$= \frac{2}{e}\left\{\frac{-axe}{n\pi x}\cos\frac{n\pi}{l} + \frac{ae^2}{n^2\pi^2}\sin\frac{n\pi}{l}x\right] - \frac{ae^2}{n\pi}\cos\frac{n\pi}{l}x + \frac{axe}{n\pi}\cos\frac{n\pi}{l}x - \frac{ae^2}{n^2\pi^2}\sin\frac{n\pi}{l}x\Big]_{e/2}^{e}$$

$$= \frac{2}{e}\left\{\frac{ae^2}{2n\pi}\cos n\pi/2 + \frac{ae^2}{n^2\pi^2}\sin\frac{n\pi}{l} - \frac{ae^2}{n\pi}(-1)^n + \frac{ae^2}{n\pi}\cos\frac{n\pi}{2} + \frac{ae^2}{n\pi}(-1) - \frac{ae^2}{2n\pi}\cos n\pi/2\right.$$

$$\left.+\frac{ae^2}{n^2\pi^2}\sin n\pi/2\right\} = \frac{2}{e}\left\{\frac{2al^2}{n^2\pi^2}sain\, n\pi/2\right\} = \begin{cases} 0 & n = even \\ \dfrac{4al}{n\pi}\sin\dfrac{2n-1}{2} & n = odd \end{cases}$$

$$\Rightarrow F(x) = \frac{4al}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\sin\frac{(2n-1)\pi}{2}.\sin\frac{(2n-1)\pi}{e}x$$

$$= \frac{4al}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\frac{(2n-1)\pi}{e}x$$

F.C.E: $F(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2ni\pi}{e}x}$

$$C_n = \frac{1}{e}\left\{\int_0^{e/2} axe^{\frac{-2ni\pi}{e}x}dx + \int_{e/2}^{e} a(e-x)e^{-\frac{2ni\pi}{e}x}dx\right\}$$

$$= \frac{1}{e}\left\{\frac{-axe}{2ni\pi}e^{-\frac{2ni\pi}{e}x} - \frac{ae^2}{4n^2\pi^2}e\right]_0^{e/2} - \frac{ae^2}{2ni\pi}e^{-\frac{2ni\pi}{e}x} + \frac{axe}{2ni\pi}e^{-\frac{2ni\pi}{e}}$$

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22) $F(x) = \begin{cases} ax & 0 \le x \le e/2 \\ a(e-x) & e/2 \le x \le e \end{cases}$

F.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi}{e}x + b_n\sin\frac{2n\pi}{e}x\right)$

$$a_0 = \frac{2}{e}\left\{\int_0^{e/2} ax\,dx + \int_{e/2}^{e} a(e-x)\,dx\right\} = \frac{2}{e}\left\{ \left. ax^2/2 \right]_0^{e/2} + \left.(aex - ax^2/2)\right]_{e/2}^{e} \right\}$$

$$= \frac{2}{e}\left\{\frac{ae^2}{8} + ae^2 - ae^2/2 + ae^2/2 - ae^2/8\right\} = 2ae$$

$$a = \frac{2}{e}\left\{\int_0^{e/2} ax\cos\frac{2n\pi}{e}x\,dx + \int_{e/2}^{e} a(e-x)\cos\frac{2n\pi}{e}x\,dx\right\}$$

$$= \frac{2}{e}\left\{\frac{axe}{2n\pi}\sin\frac{2n\pi}{e}x + \frac{ae^2}{4n^2\pi^2}\cos\frac{2n\pi}{e}x\right]_0^{e/2} + \frac{ae^2}{2n\pi}\sin\frac{2n\pi}{e}x - \frac{axl}{2n\pi}\sin\frac{2n\pi}{e}x$$

$$\left. - \frac{ae^2}{4n^2\pi^2}\cos\frac{2n\pi}{e}x\right]_{e/2}^{e}\Bigg\} = \frac{2}{e}\left\{\frac{-ae^2}{4n^2\pi^2} + \frac{ae^2(-1)^n}{4n^2\pi^2} - \frac{ae^2}{4n^2\pi^2} + \frac{ae^2(-1)^n}{4n^2\pi^2}\right\} = \frac{ae}{n^2\pi^2}\left\{(-1)^n - 1\right\}$$

$$b = \frac{2}{e}\left\{\int_0^{e/2} ax\sin\frac{2n\pi}{e}x\,dx + \int_{e/2}^{e} a(e-x)\sin\frac{2n\pi}{e}x\,dx\right\}$$

$$= \frac{2}{e}\left\{\frac{-axe}{2n\pi}\cos\frac{2n\pi}{e}x + \frac{al^2}{4n^2\pi^2}\sin\frac{2n\pi}{e}x\right]_0^{e/2} - \frac{al^2}{2n\pi}\cos\frac{2n\pi}{e}x + \frac{axe}{2n\pi}\cos\frac{2n\pi}{e}x$$

$$\left. - \frac{ae^2}{4n^2\pi^2}\sin\frac{2n\pi}{e}x\right]_{e/2}^{e}\Bigg\} = \frac{2}{e}\left\{\frac{-al^2}{4n\pi}(-1)^n - \frac{ae^2}{2n\pi} + \frac{ae^2}{2n\pi}(-1)^n + \frac{ae^2}{2n\pi} - \frac{ae^2}{4n\pi}(-1)^n\right\} = 0$$

F.C.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a\cos\frac{n\pi}{e}x$

$$a_0 = \frac{2}{e}\left\{\int_0^{e/2} ax\,dx + \int_{e/2}^{e} a(e-x)\,dx\right\} = \frac{2}{e}\left\{ \left. ax^2/2 \right]_0^{e/2} + \left.(aex - ax^2/2)\right]_{e/2}^{e} \right\}$$

$$= \frac{2}{e}\left\{\frac{ae^2}{8} + ae^2 - ae^2/2 - ae^2/2 + ae^2/8\right\} = \frac{ae}{2}$$

$$a_n = \frac{2}{e}\left\{\int_0^{e/2} ax\cos\frac{n\pi}{e}\,dx + \int_{e/2}^{e} a(e-x)\cos\frac{n\pi}{e}x\,dx\right\}$$