$$+\frac{ae^2}{4n\pi^2}e^{\frac{-2ni\pi}{e}x}\Big]_{e/2}^{e}\quad\Big\}$$

$$=\frac{1}{e}\left\{\frac{-ae^2}{4ni\pi}(-1)^n-\frac{ae^2}{4n^2\pi^2}(-1)^n+\frac{ae^2}{4n^2\pi^2}-\frac{ae^2}{2ni\pi}+\frac{ae^2}{2ni\pi}(-1)^n+\frac{ae^2}{2ni\pi}-\frac{ae^2}{4ni\pi}(-1)^n\right.$$

$$\left.-\frac{ae^2}{4n^2\pi^2}-\frac{ae^2}{4n^2\pi^2}(-1)^n\right\}=\frac{1}{e}\left\{(-1)^{n+1}\frac{ae^2}{2n^2\pi^2}+\frac{ae^2}{2n^2\pi^2}\right\}=\frac{ae}{2n^2\pi^2}\{1+(-1)^{n+1}\}$$

Nth harmonic: $F(x)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{2n\pi}{e}x-\beta_n\right)$

$$A_n^2=a_n^2+b_n^2\rightarrow A_n=a_n=\frac{ae}{n^2\pi^2}\{(-1)^n-1\}$$

$$\beta_n=tg^{-1}\left(b_n/a_n\right)=0\qquad\qquad A_0=ae$$

$$\Rightarrow F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\frac{-2ae}{(2n-1)^2\pi^2}\cos\frac{2n\pi}{e}x$$

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$$a_0=2ae\qquad\qquad a_n=\frac{ae}{n^2\pi^2}\{(-1)^n-1\}\qquad\qquad b_n=0$$

$$\Rightarrow\frac{2}{e}\left\{\int_0^{e/2}(ax)^2\,dx+\int_{e/2}^{e}a^2(e-x)^2\,dx\right\}=\frac{4a^2e^2}{2}+\sum_{n=1}^{\infty}\left(\frac{ae}{n^2\pi^2}\right)^2[(-1)^n-1]^2$$

$$\rightarrow\frac{2a^2}{e}\left\{x^3/3\Big]-\frac{(e-x)^3}{3}\Big]_{e/2}^{e}\right\}=2a^2e^2+\frac{a^2e^2}{\pi^4}\sum_{n=1}^{\infty}\frac{2}{n^4}\left((-1)^{n+1}+1\right)$$

$$\rightarrow\frac{a^2e^2}{6}-2a^2e^2=\sum_{n=1}^{\infty}\frac{2a^2e^2}{n^4\pi^4}\left(1-(-1)^n\right)$$

$$\rightarrow\frac{-11}{12}\pi^4=\sum_{n=1}^{\infty}\frac{1+(-1)^{n+1}}{n^4}$$

---

23)

$$F(x)=\begin{cases}1 & -\pi<x<0\\ \cos x & 0<x<\pi\end{cases}$$

F.S: $a_0=\dfrac{1}{\pi}\left\{\int_{-\pi}^{0}dx+\int_{0}^{\pi}\cos x\,dx\right\}=\dfrac{1}{\pi}\{\pi\}=1$

$$a=\frac{1}{\pi}\left\{\int_{-\pi}^{0}\cos nx\,dx+\underbrace{\int_{0}^{\pi}\cos x\cos nx\,dx}_{I}\right\}=\frac{1}{\pi}\left\{\left[\frac{\sin nx}{n}\right]_{-\pi}^{0}\right\}=\frac{1}{\pi}(0)=0$$

$$I=\tfrac{1}{2}\int_{0}^{\pi}(\cos(1+n)x+\cos(1-n)x)dx=\frac{1}{2}\left[\frac{\sin(1+n)x}{1+n}+\frac{\sin(1-n)x}{1-n}\right]_{0}^{\pi}=0$$

$$a=\frac{1}{\pi}\left\{\int_{-\pi}^{0}\cos x\,dx+\int_{0}^{\pi}\cos^2 x\,dx\right\}=\frac{1}{2\pi}\left\{\int_{0}^{\pi}(1+\cos 2x)dx\right\}=\frac{1}{2\pi}(\pi)=\frac{1}{2}$$

$$b_n=\frac{1}{\pi}\left\{\int_{-\pi}^{0}\sin nx\,dx+\underbrace{\int_{0}^{\pi}\cos x\sin nx\,dx}_{I_1}\right\}=\frac{1}{\pi}\left\{\left[\frac{-\cos nx}{n}\right]_{-\pi}^{0}+I_1\right\}$$

$$=\frac{1}{\pi}\left\{\frac{1}{n}\left((-1)^n-1\right)+\frac{\left(1+(-1)^n\right)n}{n^2-1}\right\}$$

$$I_1=\frac{1}{2}\int_{0}^{\pi}[Sin(1+n)x+Sin(1-n)]dx=-\frac{1}{2}\left[\frac{Cos(1+n)x}{1+n}+\frac{Cos(1-n)x}{1-n}\right]_{0}^{\pi}$$

$$=\frac{-1}{2}\left[\frac{(-1)^{n+1}}{1+n}+\frac{(-1)^{n+1}}{1-n}-\frac{1}{n+1}-\frac{1}{n-1}\right]=n\frac{1+(-1)^n}{n^2-1}=\begin{cases}0 & n=odd\\ \dfrac{2n}{n^2-1} & n=even\end{cases}$$

$$b_1=\frac{1}{\pi}\left\{\int_{-\pi}^{0}Sinxdx+\int_{0}^{\pi}CosxSinxdx\right\}=\frac{1}{\pi}\left\{\left[-Cos\right]_{-\pi}^{0}-\left[\frac{1}{4}Cos2x\right]_{0}^{\pi}\right\}$$

$$=\frac{1}{\pi}\left\{-1-1-\frac{1}{4}+\frac{1}{4}\right\}=-\frac{2}{\pi}$$

F.C.S.: $f(x)=\begin{cases}1 & 0<x<\pi\\ Cos(x-\pi) & \pi<x<2\pi\end{cases}$

$$a_0 \frac{2}{\pi}\left\{\int_0^{\pi} dx + \int_{\pi}^{2\pi} Cos(x-\pi)dx\right\} = \frac{1}{\pi}\left\{x \Big]_0^{\pi} + Sin(x-\pi) \Big]_{\pi}^{2\pi}\right\} = \frac{2}{\pi}\{\pi\} = 2$$

$$a_n = \frac{2}{\pi}\left\{\int_0^{\pi} \cos \tfrac{n}{2} x dx + \int_{\pi}^{2\pi} \cos(x-\pi)\cos \tfrac{n}{2} x dx\right\} = \frac{2}{\pi}\left\{\frac{2}{n}\sin \tfrac{n}{2} x\Big]_0^{\pi} + \frac{-1}{2}\right.$$

$$= \left[\frac{\sin\left(x-\pi+\tfrac{n}{2}x\right)}{1+\tfrac{n}{2}} + \frac{\sin\left(x-\pi-\tfrac{n}{2}x\right)}{1-\tfrac{n}{2}}\right]_{\pi}^{2\pi}$$

$$+\sin\left(1+\tfrac{n}{2}\right)x \qquad\qquad +\sin\left(1-\tfrac{n}{2}\right)x$$

$$= \frac{2}{\pi}\left\{\frac{2}{n}\sin\frac{n\pi}{2} + \frac{1}{2}\left(\frac{+\sin\left(1+\tfrac{n}{2}\right)\pi}{1+\tfrac{n}{2}} + \frac{\sin\left(1-\tfrac{n}{2}\right)\pi}{1-\tfrac{n}{2}}\right)\right\} \qquad n \neq 2$$

$$n = 2 : a_2 = \frac{2}{\pi}\left\{\int_0^{\pi} \cos x dx + \int_{\pi}^{2\pi} \cos(x-\pi)\cos x dx\right\}$$

$$-\cos 2x \qquad\qquad -\cos 2x$$

$$= \frac{2}{\pi}\left\{+\sin \Big]_0^{\pi} + \int_{\pi}^{2\pi}\left(\frac{\cos(\pi-2x)}{2} + \frac{\cos(\pi+2x)}{2}\right)dx\right\} = 0$$

$$b_n = 0$$

F.S.S

$$b_n = \frac{1}{2\pi}\left\{\int_0^{\pi} \sin \tfrac{n}{2} x dx + \int_{\pi}^{2\pi} \cos(x-\pi)\sin \tfrac{n}{2} x dx\right\}$$

$$\sin\left(\left(1+\tfrac{n}{2}\right)x-\pi\right) + \sin\left(\left(1-\tfrac{n}{2}\right)x-\pi\right)$$

$$=\frac{1}{2\pi}\left\{\left[\frac{-2}{n}\cos {}^{n}\!/\!_{2}\, x\right]_0^{\pi}+\frac{+1}{2}\left[\frac{\cos\left(1+{}^{n}\!/\!_{2}\right)x}{1+{}^{n}\!/\!_{2}}+\frac{\cos\left(1-{}^{n}\!/\!_{2}\right)x}{1-{}^{n}\!/\!_{2}}\right]_{\pi}^{2\pi}\right\}$$

$$=\frac{1}{2\pi}\left\{\frac{-2}{n}\cos\frac{n\pi}{2}+\frac{1}{2}\left[\frac{1}{1+{}^{n}\!/\!_{2}}+\frac{1}{1-{}^{n}\!/\!_{2}}-\frac{\cos\left(1+{}^{n}\!/\!_{2}\right)\pi}{1+{}^{n}\!/\!_{2}}-\frac{\cos\left(1-{}^{n}\!/\!_{2}\right)\pi}{1-{}^{n}\!/\!_{2}}\right]\right\}$$

F.X.E: $F(x)=\sum_{n=-\infty}^{\infty} C_n e^{nix}$

$$C_n=\frac{1}{2\pi}\left\{\int_0^{\pi} e^{-nix}dx+\int_{\pi}^{2\pi}\cos xe^{-nix}dx\right\}$$

$$I=\frac{1}{2\pi}\int_0^{\pi} e^{-nix}dx=\frac{1}{2\pi}\left[-\frac{1}{ni}e^{-nix}\right]_0^{\pi}=\frac{1}{2\pi}\left[\frac{1}{ni}\right]\left((-1)^n-1\right)=\frac{-1}{2ni\pi}\left((-1)^n-1\right)$$

$$I_1=\frac{1}{2\pi}\int_{\pi}^{2\pi}\cos xe^{-nix}dx=\frac{1}{2\pi}\left[e^{-nix}\sin x-nie^{-nix}\cos x+n^2 I_2\right]$$

$$I_2=\frac{1}{2\pi}\left[\frac{e^{-nix}(\sin x-ni\cos x)}{1-n^2}\right]_{\pi}^{2\pi}=\frac{1}{2\pi}\left[\frac{-ni}{1-n^2}+\frac{ni(-1)^n(-1)^n}{1-n^2}\right]$$

$$=\frac{1}{2\pi}\left[\frac{-ni}{1-n^2}+\frac{ni}{1-n^2}\right]=0$$

$$C_0=\frac{a_0}{2}=\frac{1}{2}$$

$$C_n=\frac{-1}{2nix}\left((-1)^n-1\right)\qquad n\neq 0$$

Nth harmonic: $A_0=\frac{a_0}{2}=\frac{1}{2}$

$$A_n^2=a_n^2+b_n^2$$

$$\beta_n=tg^{-1}\left({}^{b_n}\!/\!_{a_n}\right)\rightarrow\beta_n=tg^{-1}(\infty)={}^{\pi}\!/\!_{2}$$

$$\beta_1=tg^{-1}(0)=0$$

ب)

$$F(x) = 1 + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

$x = 0$
$$1 = 1 + \sum_{n=1}^{\infty} \left(a_n \cos n0 + b_n \sin n0 \right)$$

$x = \pi/2$
$$0 = \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) \rightarrow \sum_{n=1}^{\infty} \left(a_n \right) = 0$$

$$-1 = \sum_{n=1}^{\infty} \left(a_n \cos n\pi/2 + b_n \sin n\pi/2 \right)$$

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$$F(x) = \begin{cases} x^2 & 0 < x < \dfrac{3l}{4} \\ \dfrac{9l^2}{4} - \dfrac{9lx}{4} & \dfrac{3l}{4} < x < l \end{cases}$$

F.S:

$$a_0 = \frac{2}{e} \left(\int_0^{\frac{3e}{4}} x^2 dx + \int_{\frac{3e}{4}}^{l} \left(\frac{9l^2}{4} - \frac{9lx}{4} \right) dx \right) = \frac{2}{e} \left(\frac{9l^3}{64} + \frac{9l^2}{4} \left(l - \frac{3}{4} e \right) - \frac{9e}{8} \left(e^2 - \frac{9e^2}{16} \right) \right)$$

$$\Rightarrow a_0 = \frac{27}{64} e^2$$

$$a_n = \frac{2}{e} \left(\int_0^{\frac{3e}{4}} x^2 \cos \frac{2n\pi}{e} x dx + \int_{\frac{3e}{4}}^{e} \left(\frac{9e^2}{4} - \frac{9ex}{4} \right) \cos \frac{2n\pi}{e} x dx \right)$$

$$= \frac{2}{e} \left(\frac{e}{2n\pi} x^2 \sin \frac{2n\pi}{e} x + \frac{e^2}{2n^2\pi^2} x \cos \frac{2n\pi}{e} x - \frac{e^3}{4n^3\pi^3} \sin \frac{2n\pi}{e} x \right]_0^{\frac{3e}{4}}$$

$$+ \frac{9}{2} \left(\frac{e^2}{2n\pi} \sin \frac{2n\pi}{e} x - \frac{e}{2n\pi} x \sin \frac{2n\pi}{e} x - \frac{e^2}{4n^2\pi^2} \cos \frac{2n\pi}{e} x \right]_{\frac{3e}{4}}^{l}$$

$$\Rightarrow a_n = \frac{-e^2}{2n^3\pi^3} \sin \frac{3n\pi}{2} - \frac{3e^2}{8n^2\pi^2} \cos \frac{3n\pi}{2} - \frac{9e^2}{8n^2\pi^2}$$

$$b_n=\frac{2}{r}\left(\int_0^{\frac{3e}{4}}x^2\sin\frac{2n\pi}{e}x\,dx+\int_{\frac{3e}{4}}^{e}\frac{9e}{4}(e-x)\sin\frac{2n\pi}{e}x\,dx\right)$$

$$=\frac{2}{e}\left(\frac{-e}{2n\pi}x^2\cos\frac{2n\pi}{e}x+\frac{e^2}{2n^2\pi^2}x\sin\frac{2n\pi}{e}x+\frac{e^3}{4n^3\pi^3}\cos\frac{2n\pi}{e}x\right]_0^{\frac{3e}{4}}$$

$$+\frac{9}{2}\left(\frac{-e^2}{2n\pi}\cos\frac{2n\pi}{e}x+\frac{e}{2n\pi}x\cos\frac{2n\pi}{e}x-\frac{e^2}{4n^2\pi^2}\sin\frac{2n\pi}{e}x\right]_{\frac{3e}{4}}^{e}$$

$$b_n=\frac{e^2}{2n^3\pi^3}\cos\frac{3n\pi}{2}-\frac{3e^2}{8n^2\pi^2}\sin\frac{3n\pi}{2}-\frac{e^2}{2n^3\pi^3}$$

پس:

$$F(x)=\frac{a_{0.}}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi}{e}x+b_n\sin\frac{2n\pi}{e}x\right)$$

F.C.S: $b_n=0$

- برای یافتن a_n هم کافیست در a_n بدست آمده به جای n, $\frac{n}{2}$ قرار دهیم:

$$a_n=\frac{2}{e}\left(\frac{e}{n\pi}x^2\sin\frac{n\pi}{e}x+\frac{2e^2}{n^2\pi^2}x\cos\frac{n\pi}{e}x-\frac{2e^3}{n^3\pi^3}\sin\frac{n\pi}{e}x\right)_0^{\frac{3e}{4}}$$

$$+\frac{9}{2}\left(\frac{e^2}{n\pi}\sin\frac{n\pi}{e}x-\frac{e}{n\pi}x\sin\frac{n\pi}{e}x-\frac{e^2}{n^2\pi^2}\cos\frac{n\pi}{e}x\right)_{\frac{3e}{4}}^{e}$$

$$a_n=\frac{-4e^2}{n^3\pi^3}\sin\frac{3n\pi}{4}+\frac{15e^2}{2n^2\pi^2}\cos\frac{3n\pi}{4}-\frac{9e}{2n^2\pi^2}(-1)^n$$

- a_0 هم بدون تغییر باقی می ماند.

F.C.S $\Rightarrow F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi}{e}x$

F.S.S:

$$a_n = 0$$

- برای یافتن b_n، کافی است در b_n بجای n, $\frac{n}{2}$ قرار دهیم:

$$b_n = \frac{2}{e}\left(\frac{-e}{n\pi}x^2\cos\frac{n\pi}{e}x + \frac{2e^2}{n^2\pi^2}x\sin\frac{n\pi}{e}x + \frac{2e^3}{n^3\pi^3}\cos\frac{n\pi}{e}x\right)_0^{\frac{3e}{4}}$$

$$+\frac{9}{2}\left(\frac{-e^2}{n\pi}\cos\frac{n\pi}{e}x + \frac{e}{n\pi}x\cos\frac{n\pi}{e}x - \frac{e^2}{n^2\pi^2}\sin\frac{n\pi}{e}x\right)_{\frac{3e}{4}}^{e}$$

$$b_n = \frac{4e^2}{n^3\pi^3}\cos\frac{3n\pi}{4} + \frac{15e^2}{2n^2\pi^2}\sin\frac{3n\pi}{4} - \frac{4}{n^3\pi^3}$$

<u>مختلط نمايي:</u>

$$C_n = \frac{1}{2}(a_n - ib_n)$$

$$C_n = \frac{-l^2}{4n^3\pi^3}Sin\frac{3l^2}{16n^2\pi^2}Cos\frac{3n\pi}{2} - \frac{9l^2}{16n^2\pi^2} - \frac{il^2}{4n^3\pi^3}Cos\frac{3n\pi}{2} + Sin\frac{3n\pi}{2} - \frac{l^2}{4in^3\pi^3}$$

$$\to C_n = \frac{-3l^2}{16n^2\pi^2}e^{\frac{-i3n\pi}{2}} - \frac{l^2}{i4n^3\pi^3}(il.\frac{3n\pi}{2} - i^2Cos\frac{3n\pi}{2}) - \frac{9l^2}{16n^2\pi^2} - \frac{l^2}{4in^3\pi^3}$$

پس: $$C_n = \frac{-3l^2}{16n^2\pi^2}e^{\frac{-3ni\pi}{2}} - \frac{l^2}{4in^3\pi^3}e^{\frac{3ni\pi}{2}} - \frac{9l^2}{16n^2\pi^2} - \frac{l^2}{4in^3\pi^3}$$

سری مختلط نمايي $\Rightarrow f(x) = \sum_{\substack{n=\infty \\ n\neq 0}}^{\infty} C_n e^{\frac{2ni\pi}{l}x}$

$$C_0 = \frac{1}{l}\int_0^l f(x)dx = \frac{1}{l}\left(\int_0^{\frac{3l}{4}} x^2dx + \frac{9l}{4}\int_{\frac{3l}{4}}^{l}(l-x)dx\right) = \frac{27}{128}l^2 = C_0$$

پس: $$f(x) = \frac{27}{128}l^2 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} C_n e^{\frac{2ni\pi}{l}x}$$

سری دامنه مرکب: $f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{e}x - \beta_n\right)$

$$A_0 = \frac{a_0}{2} \Rightarrow A_0 = \frac{27}{128}e^2$$

$$\beta_n = \tan^{-1}\left(b/a\right) = \tan^{-1}\left(\frac{\frac{1}{2n^3\pi^3}\cos\frac{3n\pi}{2} - \frac{3}{8n^2\pi^2}\sin\frac{3n\pi}{2} - \frac{1}{2n^3\pi^3}}{\frac{-1}{2n^3\pi^3}\sin\frac{3n\pi}{2} - \frac{3}{8n^2\pi^2}\cos\frac{3n\pi}{2} - \frac{9}{8n^2\pi^2}}\right)$$

$$\Rightarrow \beta_n = \tan^{-1}\left(\frac{4\cos\frac{3n\pi}{2} - 3n\pi\sin\frac{3n\pi}{2} - 4}{-4\sin\frac{3n\pi}{2} - 3n\pi\cos\frac{3n\pi}{2} - 9n\pi}\right)$$

$$A_n^2 = a_n^2 + b_n^2 = \frac{1}{4n^6\pi^6} + \frac{9}{64n^4\pi^4} + \frac{3}{2n^5\pi^5}\sin\frac{3n\pi}{2} + \frac{1}{2n^4\pi^4}\left(\frac{27}{16} - \frac{1}{n^2\pi^2}\right)\cos\frac{3n\pi}{2}$$
$$+ \frac{1}{4n^6\pi^6} + \frac{81}{64n^4\pi^4}$$

پس:

$$A_n = \sqrt{\frac{1}{4n^6\pi^6} + \frac{9}{64n^4\pi^4} + \frac{3}{2n^5\pi^5}\sin\frac{3n\pi}{2} + \frac{1}{2n^4\pi^4}\left(\frac{27}{16} - \frac{1}{n^2\pi^2}\right)\cos\frac{3n\pi}{2} + \frac{1}{4n^6\pi^6} + \frac{81}{64n^4\pi^4}}$$

بررسی اتحاد پارسوال:

$$\frac{1}{e}\int_{-e}^{e} F^2(x)dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

$$\downarrow$$

$$\frac{2}{e}\int_{0}^{e} F^2(x)dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

$$\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right) = \frac{2}{e}\int_{0}^{e} F^2(x)dx - \frac{a_0^2}{2}$$

R.H.S $= \frac{2}{e}\left(\int_0^{\frac{3e}{4}} x^4 dx + \int_{\frac{3e}{4}}^{e} \frac{81e^2}{16}(l-x)^2 dx\right) - \frac{a_0^2}{2}$

$$= \frac{2}{5e}\times\frac{243}{1024}e^5 + \frac{81e}{8}\left(\int_{\frac{3e}{4}}^{e} e^2 dx + \int_{\frac{3e}{4}}^{e} x^2 dx - 2e\int_{\frac{3e}{4}}^{e} x dx\right) - \frac{729}{8192}e^4$$

R.H.S=$89.23e^4$

پس:

$$\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right) = 89.23e^4$$

$$\Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{4n^6\pi^6} + \frac{9}{64n^4\pi^4} + \frac{3}{2n^5\pi^5}\sin\frac{3n\pi}{2} + \frac{1}{2n^4\pi^4}\left(\frac{27}{16} - \frac{1}{n^2\pi^2}\right)\cos\frac{3n\pi}{2} + \frac{1}{4n^6\pi^6} + \frac{81}{64n^4\pi^4}\right)$$
$$= 89.23e^4$$

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25) $F(x) = \begin{cases} \cosh x & -\pi < x < 0 \\ \sinh x & 0 < x < \pi \end{cases} \Rightarrow \begin{cases} \frac{e^x + e^{-x}}{2} & -\pi < x < 0 \\ \frac{e^x - e^{-x}}{2} & 0 < x < \pi \end{cases} \quad \begin{matrix} 2\pi = 2l \\ l = \pi \end{matrix}$

F.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}x + b_n \sin\frac{n\pi}{l}x$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0}\frac{e^x + e^{-x}}{2}dx + \int_{0}^{\pi}\frac{e^x - e^{-x}}{2}dx\right] = \frac{1}{2\pi}\left(\left[e^x - e^{-x}\right] + \left[e^x + e^{-x}\right]\right)$$

$$= \tfrac{1}{2}\pi\left[\left(1 - 1 - \left(e^{-\pi} - e^{+\pi}\right)\right) + e^{\pi} + e^{-\pi} - (1+1)\right] = \frac{1}{2\pi}\left(-2 + 2e^{\pi}\right)$$

$$\Rightarrow a_0 = \frac{1}{\pi}(e^{\pi} - 1)$$

$$a_n = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(e^x + e^{-x})\cos nx dx + \int_{0}^{\pi}(e^x - e^{-x})\cos nx dx\right]$$

$$= \tfrac{1}{2\pi}\left[\int_{-\pi}^{0}\left(e^x\cos nx + e^{-x}\cos nx\right)dx + \int_{0}^{\pi}\left(e^x\cos nx - e^{-x}\cos nx\right)dx\right]$$

I_1 I_2 I_3 I_4

$$I_1 = \frac{e^x}{n}\sin nx + \frac{e^x}{n^2}\cos nx\Big]_{-\pi}^{0} - \tfrac{1}{n^x}\int_{-\pi}^{0} e\cos nx$$

$$\Rightarrow \left(1+\tfrac{1}{n^2}\right)I_1 = \frac{1}{n^2}\left(1-e^{-\pi}(-1)^n\right) \Rightarrow \left(\frac{n^2+1}{n^2}\right)I_1 = \frac{1}{n}\left(1-(-1)^n e^{-\pi}\right)$$

$$\Rightarrow I_1 = \frac{1-(-1)^n e^{-\pi}}{n^2+1}$$

$$I_2 = \frac{e^{-x}}{2}\sin nx - \tfrac{e^{-x}}{n^2}\cos nx\Big]_{-\pi}^{0} - \frac{1}{n^2}\int_{-\pi}^{0} e^{-x}\cos nx dx$$

$$\Rightarrow \left(\frac{n^2+1}{n^2}\right)I_2 = \frac{1}{n^2} - \frac{e^{+\pi}}{n^2}(-1)^n \Rightarrow I_2 = \left(\frac{n^2+1}{n^2}\right) = -\frac{1}{n^2}\left(1+(-1)^n e^{\pi}\right)$$

$$\Rightarrow I_2 = \frac{1+(-1)^n e^{\pi}}{n^2+1}$$

$$I_3 = \tfrac{e^x}{n}\sin nx + \tfrac{e^x}{n^2}\cos nx\Big]_{0}^{\pi} - \tfrac{1}{n^2}\int_{0}^{\pi} e^x\cos nx dx \Rightarrow$$

$$\Rightarrow I_3\left(1+\frac{1}{n^2}\right) = \tfrac{e^{\pi}}{n}(0) + \tfrac{e^{\pi}}{n^2}(-1)^n - \tfrac{1}{n^2} \Rightarrow I_3\left(\frac{n^2+1}{n^2}\right) = \frac{1}{n^2}\left((-1)^n e^{\pi} - 1\right)$$

$$\Rightarrow I_3 = \frac{(-1)^n e^{\pi} - 1}{n^2+1}$$