$$I_4 = \frac{e^{-x}}{n}\sin nx - \frac{e^{-x}}{n^2}\cos nx\Big]_0^{\pi} - \frac{1}{n^2}\int_0^{\pi} e^{-x}\cos nx$$

$$\left(\frac{n^2+1}{n^2}\right)I_4 = -\frac{e^{-\pi}}{n^2}(-1)^n + \frac{1}{n^2} \Rightarrow \left(\frac{n^2+1}{n^2}\right)I_4 = \frac{1}{n^2}\left(1-(-1)^n e^{-\pi}\right)$$

$$I_4 = \frac{1-e^{-\pi}(-1)^n}{n^2+1}$$

$$\Rightarrow a_n = \frac{1}{2\pi}\left(\frac{-2}{n^2+1}\right) \Rightarrow a_n = \frac{-1}{2(n^2+1)}$$

$$b_n = \frac{1}{2\pi}\int_{-\pi}^{0}(e^x + e^{-x})\sin nxdx + \int_0^{\pi}(e^x - e^{-x})\sin nxdx$$

$$I_1 = -\frac{e^x}{n}\cos nx + \frac{e^x}{n^2}\sin nx\Big]_{-\pi}^{0} - \frac{1}{n^2}\int_{-\pi}^{0} e^x \sin nxdx$$

$$\left(\frac{n^2+1}{n^2}\right)I_1 = -\frac{1}{n} + \frac{e^{-\pi}}{n}(-1)^n \Rightarrow I_1 = \left(\frac{n^2+1}{n}\right) = \left(e^{-\pi}(-1)^n - 1\right)$$

$$\Rightarrow I_1 = \frac{n(e^{-\pi}(-1)^n - 1)}{n^2+1}$$

$$I_4 = -\frac{e^{-x}}{n}\cos nx - \frac{e^{-x}}{n^2}\sin nx\Big]_{-\pi}^{0} - \frac{1}{n^2}\int_{-\pi}^{0}\sin nx(e^{-x})dx$$

$$\left(\frac{n^2+1}{n^2}\right)I_2 = \frac{-1}{n} + \frac{e^{\pi}(-1)^n}{n} \Rightarrow \left(\frac{n^2+1}{n}\right)I_2 = e^{\pi}(-1)^n - 1$$

$$\Rightarrow I_2 = \frac{n(e^{\pi}(-1)^n - 1)}{n^2+1}$$

$$I_3 = -\frac{e^x}{n}\cos nx + \frac{e^x}{n^2}\sin nx\Big]_0^{\pi} - \frac{1}{n^2}\int_0^{\pi} e^x \sin nxdx$$

$$\left(\frac{n^2+1}{n^2}\right)I_3 = -\frac{e^{\pi}}{n}(-1)^n + \frac{1}{n} \Rightarrow \left(\frac{n^2+1}{n}\right)I_3 = 1 - e^{\pi}(-1)^n \Rightarrow I_3 = \frac{n(1-e^{\pi}(-1)^n)}{n^2+1}$$

$$I_4 = -\frac{e^{-x}}{n}\cos nx - \frac{e^{-x}}{n^2}\sin nx\Big]_0^{\pi} - \frac{1}{n^2}\int_0^{\pi} e^{-x}\sin nxdx \Rightarrow$$

$$\left(\frac{n^2+1}{n^2}\right)I_4 = -\frac{e^{-\pi}}{n}(-1)^n + \frac{1}{n} \Rightarrow \frac{(n^2+1)}{n}I_4 = 1-(-1)^n e^{-\pi}$$

$$\Rightarrow I_4 = \frac{n(1-(-1)^n e^{-\pi})}{n^2+1}$$

$$b_n = (I_1 + I_2 + I_3 + I_4)\frac{1}{2\pi}$$

سری کسینوسی:

$$2l = 4\pi$$

$$F(x) = \begin{cases} \dfrac{e^{x} - e^{-x}}{2} & 0 < x < \pi \\ \dfrac{e^{x-2\pi} - e^{-x-2\pi}}{2} & \pi < x < 2\pi \end{cases}$$

$a_0 = \frac{1}{\pi}\left(e^{\pi} - 1\right)$ 　　 $b_n = 0 \rightarrow$ چون تابع زوج است.

$$a_n = \frac{1}{2\pi}\left[\int_0^{\pi} \frac{e^{x} - e^{-x}}{0} \cos \frac{n}{2} x dx + \int_{\pi}^{2\pi} \frac{e^{x-2\pi} - e^{-x-2\pi}}{0} \cos \frac{nx}{2} xdx\right]$$

F.C.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n}{2} x$

سری سینوسی فوریه:

$a_0 = \frac{1}{\pi}(e^{\pi} - 1)$ 　　 $a_n = 0$. چون تابع فرد است

$$b_n = \frac{1}{2\pi}\left[\int_0^{\pi} e^{x} - e^{-x} \Big/ 1 \sin \frac{n}{2} xdx + \int_{\pi}^{2\pi} \left(e^{x-2\pi} - e^{-x-2\pi}\right) \sin \frac{n}{2} xdx\right]$$

F.S.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin \frac{n}{2} x$

سری مختلط نمایي فوریه:

$$F(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{ni\pi}{l} x}$$

$$\Rightarrow F(x) = \sum_{n=-\infty}^{\infty} C_n e^{nix} \rightarrow \begin{cases} C_n = a_n + ib_n \\ C_{-n} = a_{-n} - ib_{-n} \end{cases}$$

با c_N بدست می آید.

جایگذاری

$$C_0 = \frac{a_0}{2} = \frac{1}{2\pi}(e^{\pi} - 1)$$

دامنه مرکب فوریه:

$$A_0=\frac{a_0}{2}=\frac{1}{2\pi}(e^{\pi}-1)$$

$tg^{-1}\left(\frac{b_n}{a_n}\right)=\beta_n$ با جایگذاری بدست می آید $A_n=\sqrt{a_n^2+b_n^2}\rightarrow$

$$F(x)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{l}x-\beta_n\right)$$

بررسی قضیه ی دیریکله:

$$F(x)=\frac{1}{2\pi}(e^{\pi}-1)+\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx$$

برای $x=0$ چون نقطه ناپیوستگی تابع است پس باید میانگین حد چپ و راست تابع در $x=0$ برابر سری فوق در $x=0$ باشد به عبارت دیگر سری به میانگین حد چپ و راست تابع در $x=0$ همگرا می باشد.

برای $x=\pi/2$ چون نقطه پیوستگی تابع است پس سری فوریه به مقدار آن در $x=\pi/2$ همگرا می باشد. (طبق قضیه دیریکله)

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26) $$F(x)=\begin{cases}x & -\pi<x<0\\ h & 0<x<\pi\end{cases}$$

$$a_0 = \frac{1}{l}\int_{c}^{c+21} F(x)dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} xdx + \int_{0}^{\pi} hdx\right) = \frac{1}{\pi}\left(-\frac{\pi^2}{2} + h\pi\right) = h - \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} x\cos nxdx + \int_{0}^{\pi} h\cos nxdx\right)$$

I_1	I_2
$+x$	$\cos nx$
-1	$\frac{1}{2}\sin nx$
0	$-\frac{1}{2}\cos nx$

$$I_1 = \left(\frac{x}{n}\sin nx + \frac{1}{n^2}\cos nx\right)_{-\pi}^{0} = \frac{1}{n^2}\left(1-(-1)^n\right)$$

$$I_2 = \int_{0}^{\pi} h\cos nxdx = \left(\frac{h}{n}\sin nx\right)_{0}^{\pi} = 0$$

$$\rightarrow a_n = \frac{1}{\pi n^2}\left(1-(-1)^n\right)$$

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} x\sin nxdx + \int_{0}^{\pi} h\sin nxdx\right)$$

I_1	I_2
	$\sin nx$
$+x$	$-\frac{1}{n}\cos nx$
-1	$-\frac{1}{n^2}\sin nx$
0	

$$I_1 = \left(\frac{-x}{n}\cos nx + \frac{1}{n^2 \sin nx}\right)_{-\pi}^{0} = \left(\frac{-\pi}{n}(-1)^n\right) = \frac{\pi}{2}(-1)^{n+1}$$

$$I_2 = \left(\frac{-h}{n}\cos nx\right)_{0}^{\pi} = \left(\frac{-h}{n}\right)\left((-1)^n - 1\right) = \frac{h}{n}\left(1-(-1)^n\right)$$

$$b_n = \frac{1}{n}(-1)^{n+1} + \frac{h}{n\pi}(1-(-1)^n)$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{\pi}{l}xdx + b_n \sin\frac{n\pi}{l}x = \left(\frac{n}{2} - \frac{\pi}{4}\right) + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$F(x)=\begin{cases} h & 0<x<\pi \\ (x-2\pi) & \pi<x<2\pi \end{cases}$$

$$a_0=\frac{1}{\pi}\left(\int_0^{\pi} h\,dx+\int_{\pi}^{2\pi}(x-2\pi)dx\right)=h-\frac{\pi}{2}$$

$$a_n=\frac{1}{\pi}\left(\int_0^{\pi} h\cos nx\,dx+\int_{\pi}^{2\pi}(x-2\pi)\cos nx\,dx\right)=\frac{1}{\pi n^2}(1-(-1)^n)$$

$$b_n=\frac{1}{\pi}\left(\int_0^{\pi} h\sin nx\,dx+\int_{\pi}^{2\pi}(x-2\pi)\sin nx\,dx\right)=\frac{1}{n}(-1)^{n+1}+\frac{h}{n\pi}(1-(-1)^n)$$

F.S.S $\Rightarrow a_n=0$ $\quad ,b_n=\frac{1}{\pi}\left(\int_0^{\pi} h\sin\frac{n}{2}x\,dx+\int_{\pi}^{2\pi}(x-2\pi)\sin\frac{n}{2}x\,dx\right)$

I_1 $\qquad$ I_2

$$I_1=\int_0^{\pi} h\sin\frac{n}{2}x\,dx=\left(\frac{-2h}{n}\cos\frac{n}{2}x\right)_0^{\pi}=\left(\frac{-2n}{n}\right)\left(\cos\frac{n\pi}{2}-1\right)$$

$+(x-2\pi)$	$\sin\frac{n}{2}x$
-1	$-\frac{2}{n}\cos\frac{n}{2}x$
0	$-\frac{4}{n^2}\sin\frac{n}{2}x$

$$I_2=\left(\frac{-2}{n}(x-2\pi)\cos\frac{n}{2}x+\frac{4}{n^2}\sin\frac{n}{2}x\right)_{\pi}^{2\pi}=$$

$$\left(\frac{-2\pi}{n}\cos\frac{n\pi}{2}-\frac{4}{n^2}\sin\frac{n\pi}{2}\right)$$

$$b_n=\cos\frac{n\pi}{2}\left(\frac{-2n}{n\pi}-\frac{2}{n}\right)+\sin\frac{n\pi}{4}\left(\frac{-4}{\pi n^2}\right)+\frac{2n}{n\pi}$$

$$F(x)=\sum_{n=1}^{\infty} b_n\sin\frac{n}{2}x\,dx$$

F.C.S $\Rightarrow b_n = 0$, $a_0 = h - \frac{\pi}{2}$

$$a_n = \frac{1}{\pi}\left(\int_0^{\pi} h\cos\frac{n}{2}x dx + \int_{\pi}^{2\pi}(x-2\pi)\cos\frac{n}{2}x dx\right)$$

$$I_1 = \int_0^{\pi} h\cos\frac{n}{2}x dx = \left(\frac{2h}{n}\sin\frac{n}{2}x\right)_0^{\pi} = \frac{2h}{n}\sin\frac{n\pi}{2}$$

I1	I2
$\cos\frac{n}{2}x$	$+(x-2\pi)$
$\frac{2}{n}\sin\frac{n}{2}x$	-1
$-\frac{4}{n^2}\cos\frac{n}{2}x$	0

$$I_2 = \left(\frac{2(x-2\pi)}{n}\sin\frac{n}{2}n + \frac{4}{n^2}\cos\frac{n}{2}x\right)_{\pi}^{2\pi} = \left(\frac{2\pi}{n}\sin\frac{n\pi}{2} - \frac{4}{n^2}\cos\frac{n\pi}{l} + \frac{4}{n^2}(-1)^n\right)$$

$$a_n = \sin\frac{n\pi}{l}\left(\frac{2h}{n\pi} + \frac{2}{n}\right) - \frac{4}{\pi n^2}\cos\frac{n\pi}{2} + \frac{4}{\pi n^2}(-1)^n$$

$$F(x) = \left(\frac{h}{2} - \frac{\pi}{4}\right) + \sum_{n=1}^{\infty} a_n \cos\frac{n}{2}xdx$$

سرى مختلط نهايي فوريه $\Rightarrow C = \frac{1}{2}(a_n - ib_n) =$

$$\frac{1}{2}\left(\frac{1}{\pi n^2}(1-(-1)^n) - \frac{i}{n}(-1)^{n+1} - \frac{hi}{n\pi}\left(1-(-1)^n\right)\right)$$

$$C_0 = \frac{a_0}{2} = \frac{h}{2} - \frac{\pi}{4} \qquad \Rightarrow F(x) = \left(\frac{h}{2} - \frac{\pi}{4}\right) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} C_n e^{nix}$$

سری دامنه مرکب فوریه $\Rightarrow A_n=\sqrt{a_n^2+b_n^2}$

$$,A_0=\frac{a_0}{2}=\frac{h}{2}-\frac{\pi}{2}$$

$$\beta_n=tg^{-1}\left(b_n/a_n\right)$$

$$F(x)=A_0+\sum_{n=1}^{\infty}A_n\left(\cos\frac{n\pi}{l}x-\beta_n\right)=\left(\frac{h}{2}-\frac{\pi}{4}\right)+\sum_{n=1}^{\infty}A_n(\cos nx-\beta_n)$$

اتحاد پارسوال

$$\frac{1}{l}\int_{-l}^{l}F^2(x)dx=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

$$\frac{1}{\pi}\left(\int_{-\pi}^{0}x^2dx+\int_{0}^{\pi}h^2dx\right)=\frac{1}{\pi}\left(\left(\frac{x^3}{3}\right)_{-\pi}^{0}+\left(h^2x\right)_0^{\pi}\right)=\frac{\pi^2}{3}+h^2 \Leftarrow$$ طرف اول

$$\frac{a_0^2}{2}=\frac{\left(n-\frac{\pi}{2}\right)^2}{2}=\frac{h^2}{2}-\frac{h\pi}{2}+\frac{\pi^2}{8}$$ عبارت اول طرف دوم

$$\sum_{n=1}^{\infty}a_n^2+b_n^2=\sum_{n=1}^{\infty}\left(\left(\frac{1}{\pi n^2}\left(1-(-1)^n\right)\right)^2+\left(\frac{1}{n}(-1)^{n+1}+\frac{h}{n\pi}\left(1-(-1)\right)^n\right)^2\right)$$

$$=\sum_{n=1}^{\infty}\frac{1}{n^2\pi^2}\left(2+2(-1)^{n+1}\right)+\frac{1}{n^2}+\frac{h^2}{n^2\pi^2}\left(2+2(-1)^{n+1}\right)+\frac{2h}{n^2\pi}(-1)^{n+1}\frac{2h}{n^2\pi}\rightarrow$$ عبارت دوم

سوال 27 و الف و ب

$$F(x)=\begin{cases} x & 0<x<\pi \\ \pi & -\pi<x<0 \end{cases} \qquad 2l=2\pi$$

سری فوریه: $$a_0=\frac{1}{l}\int_{-l}^{l}F(x)dx=\frac{1}{\pi}\int_{-\pi}^{0}\pi dx+\frac{1}{\pi}\int_{0}^{\pi}xdx=\pi+\frac{\pi}{2}=3\frac{\pi}{2}$$

$$a_n=\frac{1}{l}\int_{-l}^{l}F(x)\cos\frac{n\pi}{l}xdx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{0}\pi\cos nxdx+\frac{1}{\pi}\int_{0}^{\pi}x\cos nxdx$$

$$a_n=\frac{1}{\pi}\times\pi\left[\frac{\sin nx}{n}\right]_{-\pi}^{0}+\frac{1}{\pi}\left[\frac{x}{n}\sin nx+\frac{1}{n^2}\cos nx\right]_{0}^{\pi}=\frac{(-1)^n-1}{\pi n^2}$$

$$b_n=\frac{1}{l}\int_{-l}^{l}F(x)\sin\frac{n\pi}{l}xdx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{0}\pi\sin nxdx+\frac{1}{\pi}\int_{0}^{\pi}x\sin nxdx=\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0}+\frac{1}{\pi}\left[-\frac{x}{n}\cos nx+\frac{1}{n}\sin nx\right]_{0}^{\pi}$$

$$b_n=\left(\frac{-1}{n}+\frac{(-1)^n}{n}\right)+\frac{1}{\pi}\left[\frac{-\pi}{n}(-1)^n\right]=-\frac{1}{n}$$

$$F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{l}x+b_n\sin\frac{n\pi}{l}x\right)$$

$$F(x)=\frac{3\pi}{4}+\sum_{n=1}^{\infty}\left[\frac{(-1)^n-1}{\pi n^2}\cos nx-\frac{\sin nx}{n}\right]$$

سری سینوسی: $$F(x)=\begin{cases} x & 0<x<\pi \\ \pi & \pi<x<2\pi \end{cases} \qquad 2l=4\pi$$

$$b_n=\frac{2}{l}\int_{0}^{l}F(x)\sin\frac{n\pi}{l}x$$

$$b_n=\frac{1}{\pi}\int_{0}^{\pi}x\sin\frac{n}{2}xdx+\frac{1}{\pi}\int_{\pi}^{2\pi}\pi\sin\frac{n}{2}xdx$$

$$b_n=\frac{1}{\pi}\left[\frac{-2x}{n}\cos\frac{n}{2}x+\frac{4}{n^2}\sin\frac{n}{2}x\right]_{0}^{\pi}+\left[-\frac{2}{n}\cos\frac{n}{2}x\right]_{\pi}^{2\pi}=\frac{4}{\pi n^2}\sin\frac{n\pi}{2}-\frac{2(-1)^n}{n}$$

$$F(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi}{l}x \qquad F(x)=\sum_{n=1}^{\infty}\left(\frac{4}{\pi n^2}\sin\frac{n\pi}{2}-\frac{2}{n}(-1)^n\right)\sin\frac{n}{2}x$$

سری کسینوسی: $f(x)=\sum_{n=1}^{\infty}(a_n Cos\frac{n\pi}{l}x)+\frac{a_0}{2}$

$$a_0=\frac{1}{\pi}\int_0^{\pi}xdx+\frac{1}{\pi}\int_{\pi}^{2\pi}\pi dx=\frac{3\pi}{2}$$

$$a_n=\frac{1}{\pi}\int_0^{\pi}xCos\frac{n}{2}xdx+\frac{1}{\pi}\int_{\pi}^{2\pi}\pi Cos\frac{n}{2}xdx$$

$$a_n=\frac{1}{\pi}\left[\frac{2x}{n}Sin\frac{n}{2}x+\frac{4}{n^2}Cos\frac{n}{2}x\right]_0^{\pi}+\left[\frac{2}{n}Sin\frac{n}{2}x\right]_{\pi}^{2\pi}$$

$$a_n=\frac{4}{\pi n^2}Cos\frac{n\pi}{2}-\frac{4}{n^2} \qquad \rightarrow f(x)=\frac{3\pi}{4}+\sum_{n=1}^{\infty}(\frac{4}{\pi n^2}Cos\frac{n\pi}{2}-\frac{4}{n^2})Cos\frac{n}{2}x$$

سری مختلط نمایی: $f(x)=\sum_{n=-\infty}^{\infty}C_n e^{\frac{in\pi}{l}x}$

$$C_n=\frac{a_n-ib_n}{2}=\frac{(-1)^n-1}{2\pi n^2}+\frac{i}{2n}$$

$$n\neq 0, C_0=\frac{a_0}{2}$$

$$f(x)=\frac{3\pi}{4}+\sum_{n=-\infty}^{\infty}(\frac{(-1)^n-1}{2\pi n^2}-\frac{i}{2n})e^{inx}$$

سری هارمونیک: $F(x)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{l}x-\beta_n\right)$

$$A_0=\frac{a_0}{2}=\frac{3\pi}{4} \qquad ,A_n=\sqrt{a_n^2+b_n^2}=\sqrt{\frac{4}{\pi^2(2n-1)^2}+\frac{1}{n^2}}$$

$$\beta_n=\tan^{-1}\left(\frac{b_n}{a_n}\right)=\tan^{-1}\left(\frac{\pi n^2}{n(1-(-1)^n)}\right)=\beta_n$$

$$\rightarrow F(x)=3\pi/4+\sum_{n=1}^{\infty}\sqrt{\frac{4}{\pi^2(2n-1)^2}+\frac{1}{n^2}}\cos(nx-\beta_n)$$

سوال 28

$$F(x)=\begin{cases} x^2\sin x & 0<x<\pi \\ x & -\pi<x<0 \end{cases} \qquad 2l=2\pi \to l=\pi$$

$$a_0=\frac{1}{\pi}\left\{\int_{-\pi}^{0}xdx+\int_0^{\pi}x^2\sin xdx\right\}=\frac{1}{\pi}\left\{\frac{x^2}{2}\Big|_{-\pi}^{0}+\left(x^2\cos x+2x\sin x+2\cos x\right)_0^{\pi}\right\}=\frac{\pi}{2}-\frac{4}{\pi}$$

$$a_n=\frac{1}{\pi}\left\{\int_{-\pi}^{0}x\cos nxdx+\int_0^{\pi}x^2\sin x\cos nxdx\right\}=\frac{1}{\pi}\left\{\frac{x\sin nx}{n}+\frac{\cos nx}{n^2}\Big|_{-\pi}^{0}+\frac{1}{2}\left(\frac{-x^2\cos(n+1)x}{n+1}+\right.\right.$$

$$\left.\left.\frac{2a\sin(n+1)x}{(n+1)^3}+\frac{2\cos(n+1)x}{(n+1)}\Big|_0^{\pi}+\frac{1}{2}\left(\frac{-x^2\cos(n-1)x}{n-1}+\frac{2x\sin(n-1)x}{(n-1)}+\frac{2\cos(n-1)x}{(n-1)}\Big|_0^{\pi}\right.\right\}$$

$$\to a_m=\frac{1}{\pi}\left\{\frac{1}{n^2}-\frac{(-1)^n}{n^2}+\frac{1}{2}\left(\frac{2(-1)^{n+1}}{(n+1)^3}-\frac{2}{(n+1)^3}+\frac{\pi^2(-1)^n}{n+1}\right)+\frac{1}{2}\left(\frac{2(-1)^{n-1}}{(n-1)^3}-\frac{2}{(n-1)^3}-\frac{\pi^2(-1)^{n-1}}{(n-1)}\right.\right.$$

$n\neq\pm 1$

$$a_{-1}=\frac{2}{\pi}-\frac{\pi}{4}$$

$$b_n=\frac{1}{\pi}\left\{\int_{-\pi}^{0}x\sin nxdx+\int_0^{\pi}x^2\sin x\sin nxdx\right\}=\frac{1}{\pi}\left\{x\sin nxdx+\int_0^{\pi}\frac{x^2}{2}(\cos)n-1)x-\cos(n+1)x)dx\right.$$

$$-\frac{1}{\pi}\left\{\frac{-x\cos nx}{n}+\frac{\sin nx}{n}\Big|_{-\pi}^{0}+\frac{1}{2}\left(\frac{x^2\sin(n-1)x}{n-1}+\frac{2x\cos(n-1)x}{(n-1)^2}-\frac{2\sin(n-1)x}{(n-1)^3}\Big|_0^{\pi}-\frac{1}{2}\left\{\frac{x^2\sin(n-1)x}{n-1}\right.\right.\right.$$

$$\left.+\frac{2x\cos(n-1)x}{(n-1)^2}-\frac{2\sin(n-1)x}{(n-1)^3}\right)_0^{\pi}\Bigg\}$$

$$\to b_n=(-1)^n\left(\frac{1}{(n+1)^2}-\frac{1}{(n-1)^2}-\frac{1}{n}\right) \qquad \begin{cases} b_1=\frac{3}{4}+\frac{\pi^2}{6} \\ b_{-1}=\left(\frac{3}{4}+\frac{\pi^2}{6}\right) \end{cases}$$

$n\neq\pm 1$

$$C_n=\frac{1}{2}(a_n-ib_n)$$

$$C_0=\frac{a_0}{2}=\frac{\pi}{8}-\frac{2}{\pi}$$

$$C_n=\frac{1}{2}\left(\left(\frac{2}{\pi}\right)-\frac{\pi}{4}-i\left(3/4+\pi^2/6\right)\right) \qquad F(x)=C_0+Ce^{ix}+Ce^{-ix}+\sum_{\substack{n=-\infty \\ n\neq 0,\pm 1}}^{\infty}c_n e^{inx}$$