$$C_1=\frac{1}{2}\left(\left(\frac{2}{\pi}-\frac{\pi}{4}\right)+i\left(3/4+\frac{\pi^2}{6}\right)\right)$$

$$A_n=\sqrt{a_n^2+b_n^2}$$
$n\neq 0,1$

$$A_0=a_{0/2}=\frac{\pi}{8}-\frac{2}{\pi}$$

$$F(x)=A_0+A_1\cos(x-\beta_1)+\sum_{n=2}^{\infty}A_n\cos(n_n-\beta_n)$$

$$A_1=\sqrt{a_1^2+b_1^2}$$

$$\beta_n=\tan^{-1}\left(b_n/a_n\right)$$

$$F(x)=\frac{\pi}{8}-\frac{2}{\pi}+A_1\cos(x-\beta_1)+\sum_{n=2}^{\infty}A_n\cos(n_x-\beta_n)$$

برای نوشتن سری سینوسی و کسینوسی باید تابع جدید تشکیل دهیم.

$$F(x)=\begin{cases}x^2\sin x & 0<x<\pi\\ x & -\pi<x<0\end{cases}$$

$$F(x)=\begin{cases}x^2\sin x & 0<x<\pi\\ (x-2\pi) & \pi<x<2\pi\end{cases}$$

$$a_0=\frac{1}{n}\left\{\int_0^{\pi}x^2\sin x\,dx+\int_{\pi}^{2\pi}(x-2\pi)dx\right\}=\pi/2-4/\pi$$

$$a_n=\frac{1}{\pi}\left\{\int_0^{\pi}x^2\sin x\cos\left(nx/2\right)dx+\int_{\pi}^{2\pi}(x-2\pi)\cos\left(nx/2\right)dx\right\}$$

$$b_n=\frac{1}{\pi}\left\{\int_0^{\pi}x^2\sin x\sin\left(nx/2\right)dx+\int_{\pi}^{2\pi}(x-2\pi)\sin\left(nx/2\right)\right\}$$

اتحاد پارسوال:

$$\frac{1}{\pi}\left\{\int_{-\pi}^{0}x^2dx+\int_0^{\pi}(x^2\sin x)^2dx\right\}=\frac{a_0^2}{2}+\sum a_n^2+b_n^2$$

سوال 29) $F(x)=\begin{cases}0 & -\pi<x<0\\ x+1 & 0<x<\frac{\pi}{2}\\ 2x & \frac{\pi}{2}<x<\pi\end{cases}$ $\qquad 2\pi=2l\Rightarrow \pi=1$

F.S $\quad a_0={}^1\!/_\pi\left(\int_{-\pi}^{0}0dx+\int_{0}^{\frac{\pi}{2}}(x+1)dx+\int_{\frac{\pi}{2}}^{\pi}2xdx\right)=\frac{1}{\pi}\left(\frac{x^2}{2}+x\Big]_0^{\frac{\pi}{2}}+x^2\Big]_{\frac{\pi}{2}}^{\pi}\right)$

$$={}^1\!/_\pi\left(\frac{\pi^2}{8}+\frac{\pi}{2}+\pi^2-\frac{2\pi^2}{8}\right)={}^1\!/_\pi\left(\frac{7}{8}\pi^2+\frac{\pi}{2}\right)=\frac{1}{2}+\frac{7}{8}\pi$$

$$a_n={}^1\!/_\pi\left(\int_0^{\frac{\pi}{2}}(x+1)\cos nxdx+\int_{\frac{\pi}{2}}^{\pi}2x\cos nxdx\right)$$

$$={}^1\!/_\pi\left(\frac{(x+1)}{n}\sin nx+{}^1\!/_n\cos nx\Big]_0^{\frac{\pi}{2}}+\frac{2x}{x}\sin nx+\frac{2}{n}\cos nx\Big]_{\frac{\pi}{2}}^{\pi}\right)$$

$$={}^1\!/_\pi\left(\frac{\frac{\pi}{2}+1}{n}\sin\frac{n\pi}{2}+\frac{1}{n^2}\cos\frac{n\pi}{2}-\frac{1}{n^2}+\frac{2}{n^2}(-1)^n-\frac{\pi}{n}\sin\frac{n\pi}{2}-\frac{2}{n^2}\cos\frac{n\pi}{2}\right)$$

$$={}^1\!/_\pi\left(-{}^1\!/_{n^2}+{}^2\!/_{n^2}(-1)^n-{}^1\!/_{n^2}\cos\frac{n\pi}{2}+\frac{\left(1-\frac{\pi}{2}\right)}{n}\sin\frac{n\pi}{2}\right)$$

$$\begin{cases}{}^1\!/_\pi\left(-{}^1\!/_{n^2}+{}^2\!/_{n^2}-{}^1\!/_{n^2}(-1)^n+0\right) & n=even, m=1,2,\ldots\\ {}^1\!/_\pi\left(-{}^1\!/_{n^2}-\frac{2}{n^2}+\frac{\left(1-\frac{\pi}{2}\right)}{n}(-1)^{m+1}\right) & n=odd, m=1,2,\ldots\end{cases}$$

$$\Rightarrow a_n=\begin{cases}{}^1\!/_\pi\left({}^1\!/_{n^2}-{}^1\!/_{n^2}(-1)^m\right) & n=even, m=1,2,\ldots\\ {}^1\!/_\pi\left(-{}^3\!/_{n^2}+\frac{\left(1-\frac{\pi}{2}\right)}{n}(-1)^{m+1}\right) & n=odd, m=1,2,\ldots\end{cases}$$

$$b_n = \frac{1}{\pi}\left(\int_0^{\pi/2}(x+1)\sin nx\,dx + \int_{\frac{\pi}{2}}^{\pi} 2x\sin nx\,dx\right)$$

$$= \frac{1}{\pi}\left(-\frac{(x+1)}{n}\cos nx + \frac{1}{n^2}\sin nx\Big]_0^{\frac{\pi}{2}} - \frac{2x}{n}\cos nx + \frac{2}{n}\sin nx\Big]_{\frac{\pi}{2}}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(\frac{-\frac{\pi}{2}-1}{n}\cos\frac{n\pi}{2} + \frac{1}{n^2}\sin\frac{n\pi}{2} + \frac{1}{n} - \frac{2\pi}{n}(-1)^n + \frac{\pi}{n}\cos\frac{n\pi}{2} - \frac{2}{n^2}\sin\frac{n\pi}{2}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{n} + \frac{\frac{\pi}{2}-1}{n}\cos\frac{n\pi}{2} - \frac{1}{n^2}\sin\frac{n\pi}{2} - \frac{2\pi}{n}(-1)^n\right)$$

F.C.S: $4\pi = 2l \Rightarrow l = 2\pi$

چون تابع مربوطه زوج است. $b_n = 0$

$$a_0 = \left(\frac{1}{2\pi}\times 2\right)\left(\int_0^{\frac{\pi}{2}}(x+1)dx + \int_{\frac{\pi}{2}}^{\pi} 2x\,dx\right) = \frac{1}{2} + \frac{7}{8}$$

$$a = \left(\frac{1}{2\pi}\times 2\right)\left(\int_0^{\frac{\lambda}{2}}(x+1)\cos\frac{n}{2}x\,dx + \int_{\frac{\pi}{2}}^{\pi} 2x\cos\frac{n}{2}x\,dx\right)$$

$$= \frac{1}{\pi}\left(\frac{2(x+1)}{n}\sin\frac{n}{2}x + \frac{4}{n^2}\cos\frac{n}{2}x\Big]_0^{\frac{\pi}{2}} + \frac{4x}{n}\sin\frac{n}{2}x + \frac{8}{n^2}\cos\frac{n}{2}x\Big]_{\frac{\pi}{25}}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi+2}{n}\sin\frac{n\pi}{4} + \frac{4}{n^2}\cos\frac{n\pi}{4} - \frac{4}{n^2} + \frac{4\pi}{n}\sin\frac{n\pi}{2} + \frac{8}{n^2}\cos\frac{n\pi}{2} - \frac{2\pi}{n}\sin\frac{n\pi}{4}\right.$$
$$\left. - \frac{8}{n^2}\cos\frac{n\pi}{2}\right)$$

$$\Rightarrow a_n = \frac{1}{\pi}\left(\frac{2-\pi}{n}\sin\frac{n\pi}{4} - \frac{4}{n^2}\cos\frac{n\pi}{4} - \frac{4}{n^2} + \frac{4\pi}{n}\sin\frac{n\pi}{2} + \frac{8}{n^2}\cos\frac{n\pi}{2}\right)$$

F.C.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a\cos\frac{nx}{2}$

F.S.S: $4\pi = 2l \Rightarrow l = 2\pi$

$$a_0 = \frac{1}{2\pi} \times 2\left(\int_0^{\pi/2} (x+1)dx + \int_{\frac{\pi}{2}}^{\pi} 2x dx \right) = 1/2 + 7/8\,\pi$$

چون تابع مربوطه فرد است. $a_n = 0$

$$b_n = 1/\pi \left(\int_0^{\pi/2} (x+1)\sin\frac{nx}{2} dx + \int_{\frac{\pi}{2}}^{\pi} 2x \sin\frac{nx}{2} dx \right)$$

$$= 1/\pi \left(\left. \frac{-2(x+1)}{n}\cos\frac{n}{2}x + 4/n^2 \sin\frac{n}{2}x \right]_0^{\frac{\pi}{2}} + \left(\left. \frac{-4x}{n}\cos\frac{n}{2}x + \frac{8}{n^2}\sin\frac{n}{2}x \right]_{\frac{\pi}{2}}^{\pi} \right) \right)$$

$$= 1/\pi \left(\frac{-\pi-2}{n}\cos\frac{n\pi}{4} + 4/n^2 \sin\frac{n\pi}{4} + \frac{2}{n} - \frac{4\pi}{n}\cos\frac{n\pi}{2} + 8/n^2 \sin\frac{n\pi}{2} + \frac{2\pi}{n}\cos\frac{n\pi}{4} \right.$$
$$\left. - 8/n^2 \sin n\pi/4 \right)$$

$$\Rightarrow b_n = 1/\pi \left(\frac{\pi-2}{n}\cos\frac{n\pi}{4} - 4/n^2 \sin\frac{n\pi}{4} + \frac{2}{n} - \frac{4\pi}{n}\cos\frac{n\pi}{2} + 8/n^2 \sin\frac{n\pi}{2} \right)$$

F.S.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b \sin\frac{n}{2}x$

F.C.E: $F(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{ni\pi}{l}x}, C_0 = \frac{a_0}{2}$

$C_n = 1/2l \int_c^{c+21} F(x) e^{\frac{-ni\pi}{l}} dx$ $\quad 2\pi = 2l \Rightarrow l = \pi$

$$C_n = \frac{1}{2\pi}\left(\int_{-\pi}^{0}(0)e\,dx + \int_{0}^{\frac{\pi}{2}}(x+1)e\,dx + \int_{\frac{\pi}{2}}^{\pi}2xe\,dx\right)$$

$$= \frac{1}{2\pi}\left(\frac{(x+1)}{n}ie^{-nix} + \frac{1}{n}e\Big]_0^{\frac{\pi}{2}} + \frac{2ix}{n}e^{-nix} + \frac{2}{n}e^{-nix}4\Big]_{\frac{\pi}{2}}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{\left(\frac{\pi}{2}+1\right)}{n}ie^{-\frac{nix}{2}} + \frac{1}{n^2}e^{-\frac{nix}{2}} - \frac{i}{n} - \frac{1}{n} + \frac{2i\pi}{n}(-1) + \frac{2}{n}(-1) - \frac{i\pi}{n}e + \frac{2}{n}\right)$$

$$\Rightarrow \begin{cases} C_n = \dfrac{1}{2\pi}\left(\dfrac{i-\frac{\pi}{2}i}{n}e^{-\frac{ni\pi}{2}} - \dfrac{1}{n^2}e^{-\frac{ni\pi}{2}} - \dfrac{i}{n} - \dfrac{1}{n^2} + \dfrac{2i\pi}{n}(-1)^n + \dfrac{2}{n^2}(-1)^n\right) \\ C_0 = \dfrac{a_0}{2} = \dfrac{1}{4} + \dfrac{7}{16}\pi \end{cases}$$

$$\Rightarrow F(x) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} C_n e^{nix} + \frac{a_0}{2}$$

دامنه ی مرکب فوریه :

$$F(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{l} - \beta_n\right)$$

$$tg^{-1}\left(\frac{b_n}{a_n}\right) = \beta_n, A_n = \sqrt{a_n^2 + b_n^2}, A_0 = \frac{a_0}{2} = \frac{1}{4} + \frac{7}{16}\pi$$

$$\Rightarrow F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n_x - \beta_n)$$

بررسی قضیه دیریکله :

$$F(0^-)=0, F(0^+)=1 \Rightarrow \frac{1+0}{2}=\frac{1}{2}$$

$$F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$\xrightarrow{x=0} \quad \frac{1}{2}=\frac{1}{4}+\frac{7}{16}\pi+\sum_{n=1}^{\infty} a_n(1) \Rightarrow \frac{1}{4}-\frac{7}{16}\pi=\sum_{n=1}^{\infty} a_n$$

$$\longrightarrow$$

$$x=\frac{\pi}{2} F\left(\frac{\pi^-}{2}\right)\frac{\pi}{2}+1, F\left(\frac{\pi^+}{2}\right)=\pi \Rightarrow \frac{\pi+\frac{\pi}{2}+1}{2}=\frac{3\pi}{4}+\frac{1}{2}\frac{3\pi}{4}+\frac{1}{2}=\frac{1}{4}+\frac{7}{16}\pi+$$
$$\sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{2}+b_n \sin\frac{n\pi}{2}$$

سوال 30)

$$F(x)=\begin{cases} 0 & 0<x<h-a \\ 1 & h-a<x<h+a \\ 0 & h+a<x<\pi \end{cases} \qquad 21=\pi \Rightarrow l=\frac{\pi}{2}$$

F.S: $$a_0=\frac{2}{\pi}\left(\int_0^{h-a} 0dx+\int_{h-a}^{h+a}(1)dx+\int_{h+a}^{\pi}0dx\right)=\frac{2}{\pi}(h+a-h+a)=\frac{4a}{\pi}$$

$$a_n=\frac{2}{\pi}\left(\int_{h-a}^{h+a}\cos 2nxdx\right)=\frac{2}{\pi}\left(\frac{1}{2n}\sin^2 nx\right)\Big]_{h-a}^{h+a}$$

$$\Rightarrow a_n=\frac{1}{\pi n}(\sin 2n(h+a)-\sin 2n(h-a))$$

اگر : $h\to 0: a_n=\frac{1}{\pi n}(\sin 2na+\sin 2na)=\frac{2}{\pi n}\sin 2na$

$$-\frac{1}{\pi n}(\cos 2n(h+a)-\cos(2n)(h-a))$$

اگر : $h\to 0=\frac{-1}{\pi n}(\cos 2na-\cos 2na)=0 \Rightarrow b_n=0$

F.C.S: $2\pi=2l \Rightarrow l=\pi$

چون تابع مورد نظر زوج است. $b_n = 0$

$$a_0 = \frac{1}{\pi} \times 2\left(0 + \int_{h-a}^{h+a} dx + 0\right) = \frac{2}{\pi}(h + a - h + a) = \frac{49}{\pi}$$

$$a_n = \frac{2}{\pi}\left(\int_{h-a}^{h+a} \cos nxdx\right) = \frac{2}{\pi}\left(\frac{1}{n}\sin nx\right)\Big]_{h-a}^{h+a}$$

$$\Rightarrow a_n = \frac{2}{n\pi}(\sin n(h+a) - \sin n(h-a))$$

اگر : $h \to 0 : a_n = \frac{2}{n\pi}(\sin na + \sin na) = \frac{4}{n\pi}(\sin na)$

F.C.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx = \frac{2a}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin na}{n}\cos nx$

F.S.S: $a_n = 0$ چون تابع فرد است. , $a_0 = \frac{4a}{\pi}$

$$b_n = \frac{2}{\pi}\left(0 + \int_{h-a}^{h+a} \sin xdx + 0\right) = \frac{-2}{\pi}(\cos x)\Big]_{h-a}^{h+}$$

$$\Rightarrow b_N = \frac{-2}{n\pi}(coa_N(h+a) - \cos_N(h-a))$$

اگر : $h \to 0 : b_n = \frac{-2}{n\pi}(\cos na - coana) = 0$

F.S.S: $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin nx$

F.C.E: $F(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{ni\pi}{l}x}, \pi = 21 \Rightarrow l = \frac{\pi}{2}$

$$C_n = \frac{1}{\pi}\left(\int_{h-a}^{h+a} e^{-2nix}dx\right) = \frac{1}{\pi}(-2ni)e^{-2nix}\Big]_{h-a}^{h+a}$$

$$= \frac{-2ni}{\pi}\left(e^{-2nih-2nia} - e^{-2nih+2nia}\right) \qquad h \to 0 :$$

$$C_n = \frac{2ni}{\pi}(e^{2nia} - e^{-2nia}) \times \frac{2i}{2i} = \frac{-4n}{\pi}\sin 2na$$

دامنه ی مرکب:

$$F(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{l}x-\beta_n\right)$$

$$tg^{-1}\left(\frac{b_n}{a_n}\right)=\beta_n, A_n=\sqrt{a_n^2+b_n^2}$$

$$F(x)=\frac{2a}{\pi}+\sum_{n=1}^{\infty}A_n\cos(2nx-\beta_n)$$

اگر $h\to 0: tg^{-1}\left(0/a\right)=0=\beta_n, A_n=a_n=\frac{2}{n\pi}\sin 2na$

$$\Rightarrow F(x)=2a/\pi+\sum_{n=1}^{\infty}\frac{2}{n\pi}\sin 2na.\cos 2nx$$

اتحاد پارسوال:

$$\frac{1}{l}\int_{-l}^{l}F^2dx=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

$$\frac{1}{l}\int_{-l}^{l}F^2dx=\frac{2}{\pi}\left(\int_{h-a}^{h+a}(-1)^2dx\right)=\frac{2}{\pi}(h+a-h+a)=\frac{4a}{\pi}$$

$$a_0^2=\frac{16a^2}{\pi^2}\Rightarrow\frac{a_0^2}{2}=\frac{8a^2}{\pi^2}$$

$$h\to 0\Rightarrow a_n=\frac{2}{n\pi}\sin 2na\Rightarrow a_n^2=\frac{4}{n^2\pi^2}\sin^2 2na$$

$$\Rightarrow\frac{4a}{\pi}=\frac{8a^2}{\pi^2}+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\sin^2 2na$$

---

31) سوال

$$\cos^4 x=\left(\frac{1+\cos x}{2}\right)^2=\frac{1+\cos^2 2x+2\cos 2x}{4}=\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1+\cos 4x}{8}$$

$$\cos^4 x = \frac{1}{8}\cos 4x + \tfrac{3}{8} + \frac{1}{2}\cos 2x \qquad 2\pi = 2l \Rightarrow l = \pi$$

F.S: $$a_0 = \frac{1}{\pi}\left(\int_0^{2\pi}\left(\frac{1}{8}\cos 4x + \frac{3}{8} + \frac{1}{2}\cos 2x\right)\right)dx =$$

$$\frac{1}{\pi}\left(\frac{1}{32}\cos in4x + \frac{3}{8}x + \frac{1}{4}\sin 2x\right]_0^{2\pi} = \frac{1}{\pi}\left(\frac{6\pi}{8}\right) \Rightarrow a_0 = \frac{3}{4}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{1}{8}\cos 4x + \frac{3}{8} + \frac{1}{2}\cos 2x\right)\cos xdx$$

$$I_1 = \tfrac{1}{8}\int_0^{2\pi}\cos 4x\cos nxdx = \frac{1}{16}\int_0^{2\pi}(\cos(n+4)x + \cos(n-4)x)dx$$

$$= \tfrac{1}{16}\left(\frac{\sin(n+4)x}{n+4} + \frac{\sin(n-4)x}{n-4}\right]_0^{2\pi} = 0$$

$$I_2 = \tfrac{3}{8}\int_0^{2\pi}\cos nxdx = \frac{3}{8}\left(\tfrac{1}{n}\sin nx\right)\Big]_0^{2\pi} = 0$$

$$I_3 = \frac{1}{2}\int_0^{2\pi}(\cos 2x\cos nx)dx = \tfrac{1}{4}\int_0^{2\pi}(\cos(n+2)x + \cos(n-2)x)dx$$

$$= \tfrac{1}{4}\left(\frac{\sin(n+2)x}{n+2} + \frac{\sin(n-2)}{n-2}x\right)\Big]_0^{2\pi} = 0 \Rightarrow a_n = 0$$

$$b_n = \tfrac{1}{\pi}\int_0^{2\pi}\left(\frac{1}{8}\cos 4x + \tfrac{3}{8} + \frac{1}{2}\cos 2x\right)\sin nxdx$$

$$I_1 = \tfrac{1}{8}\int_0^{2\pi}(\sin(n+4)x + \sin(n-4)x)dx$$

$$= \tfrac{1}{16}\left(\frac{-\cos(n+4)x}{n+4} - \frac{\cos(n-4)}{n-4}\right)\Big]_0^{2\pi} = \frac{1}{16}\left(\frac{-1}{n+4} - \frac{1}{n-4} + \frac{1}{n+4} + \frac{1}{n-4}\right) = 0$$

$$I_2 = \tfrac{3}{8}\int_0^{2\pi}\sin nxdx = \frac{-3}{8n}\cos nx\Big]_0^{2\pi} = 0$$

$$I_3 = \frac{1}{2}\int_0^{2\pi}\cos 2x\sin nx = \tfrac{1}{4}\int_0^{2\pi}(\sin(n+2)x + \sin(n-2)x)dx$$

$$= \tfrac{1}{4}\left(\frac{-\cos(n+2)x}{n+2} - \frac{\cos(n-2)}{n-2}x\right)\Big]_0^{2\pi} = \frac{1}{4}\left(\frac{-1}{n+2} - \frac{1}{n-2} + \frac{1}{n+2} + \frac{1}{n-2}\right) = 0$$

$\Rightarrow b_n = 0$ نتیجه می گیریم که → $a_2 = \frac{1}{2}, a_4 = \tfrac{1}{8}$

F.S: $F(x)=\cos^4 x=\frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x$

F.C.S: $4\pi=2l \Rightarrow l=2\pi$

$$a_0=\tfrac{1}{2\pi}\times 2\left(\int_0^{2\pi}\left(\tfrac{1}{8}\cos 4x+\tfrac{3}{8}+\frac{1}{2}\cos 2x\right)dx \Rightarrow a_0=\tfrac{3}{4}\right.$$

$$a_n=\tfrac{1}{\pi}\left(\int_0^{2\pi}\left(\tfrac{1}{8}\cos 4x+\tfrac{3}{8}+\frac{1}{2}\cos 2x\right)\cos\frac{n}{2}xdx\right.$$

$$I_1=\tfrac{1}{8}\int_0^{2\pi}\cos 4x\cos\frac{n}{2}x=\begin{cases}0 & n\neq 8\\ \frac{\pi}{8} & n=8\end{cases}$$

$$I_2=\tfrac{3}{8}\int_0^{2\pi}\cos\frac{n}{2}xdx=\frac{3}{4n}\sin\frac{n}{2}x\Big]_0^{2\pi}=0$$

$$I_3=\frac{1}{2}\int_0^{2\pi}\left(\cos 2x\cos\frac{n}{2}x\right)dx=\begin{cases}0 & n\neq 4\\ \frac{\pi}{2} & n=4\end{cases}$$

$$\Rightarrow a_8=1, a_4=1$$

$$b_n=\tfrac{1}{\pi}\int_0^{2\pi}\left(\tfrac{1}{8}\cos 4x+\tfrac{3}{8}+\tfrac{1}{2}\cos 2x\right)\sin\tfrac{n}{2}xdx$$

$I_1=\tfrac{1}{8}\int_0^{2\pi}\cos 4x\sin\tfrac{n}{2}x=0, b_n=0$. چون تابع زوج است

F.C.S: $F(x)=\frac{3}{8}+\sum_{n=1}^{\infty}a_n\cos\frac{n}{2}x=\tfrac{3}{8}+\tfrac{1}{8}\cos 4x+\frac{1}{2}\cos 2x$

F.S.S: $a_n=0$. چون تابع فرد است $a_0=\tfrac{3}{4}$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}\left(\tfrac{1}{8}\cos 4x+\tfrac{3}{8}+\tfrac{1}{2}\cos 2x\right)\sin\frac{n}{2}xdx$$

$$I_1=\tfrac{1}{8}\int_0^{2\pi}\cos 4x\sin\frac{n}{2}xdx=\tfrac{1}{16}\int_0^{2\pi}\left(\sin\left(\frac{n}{2}+4\right)x+\sin\left(\frac{n}{2}-4\right)x\right)dx$$

$$=\tfrac{1}{16}\left(\frac{-\cos\left(\tfrac{n}{2}+4\right)x}{\tfrac{n}{2}+4}-\frac{\cos\left(\tfrac{n}{2}-4\right)x}{\tfrac{n}{2}-4}\right)\Bigg]_0^{2\pi}=0$$