$$\begin{cases}\beta = 0 \\ \beta_m = \dfrac{m\Pi}{a}\end{cases} \longrightarrow \beta_m = \frac{m\pi}{a}, m = 0,1,2,\ldots \Rightarrow M(x) = A_2 \cos\frac{m\pi}{a}x$$

$$-\frac{N''}{N} = -\wp^2 \to N + \wp^2 N = 0 \to N(y) = B_1 \sin\wp y + B_2 \cos\wp y$$

$$N(0) = 0 \Rightarrow B_2 = 0 \to B_1 \neq 0$$

$$N(b) = 0 \Rightarrow B_1 \sin\wp b = 0 \Rightarrow \sin\wp b = \sin n\pi \Rightarrow \wp_n = \frac{n\pi}{b}, n = 1,2,3,\ldots N(y) = B_1 \sin\frac{n\pi}{b}y$$

$$-\frac{P''}{C^2 P} = -\lambda^2 \Rightarrow P'' + c^2\lambda^2 p = 0 \Rightarrow P(t) = D_1 \cos c\lambda t + D_2 \sin c\lambda t$$

$$-\beta^2 - \wp^2 = -\lambda^2 \Rightarrow \lambda = \sqrt{\beta^2 + \wp^2} \to \lambda_{nm} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

پس: $U(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y\left(A_{mn}\cos c\lambda_{mn} + B_{mn}\sin c_{mn}\lambda t\right)$

$$U_t(x,y,0) = 0 \Rightarrow B_{mn} = 0$$

$$U(x,y,0) = \cos y \sin x = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} A_{mn} \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$A_{nm} = \frac{4}{ab}\int_0^b\int_0^a \cos y \sin x \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y\,dxdy$$

پاسخ 3 قسمت ج :

$$U(x,y,t) = M(x).N(y).P(t)$$

$$\Rightarrow \frac{M''}{M} + \frac{N''}{N} = \frac{P''}{C^2 P} = -\lambda^2$$

$$-P + \lambda^2 C^2 P = 0 \to P(t) = A_1 \cos c\lambda + A_2 \sin c\lambda t$$

$$-\frac{M''}{M} = -\beta^2 \Rightarrow M'' + \beta^2 M = 0 \Rightarrow M(x) = B_1 \sin\beta x + B_2 \cos\beta x$$

$$M(0) = 0 \to B_2 = 0 \to B_1 \neq 0$$

$$M''(x) = \beta(B_1 \cos\beta x - B_2 \sin\beta x) \xrightarrow{B_2 = 0} M''(x) = \beta B_1 \cos\beta x$$

$$\cos\beta a$$

$$M''(a) = 0 \Rightarrow \beta B = 0 \Rightarrow \begin{cases}\beta = 0 \\ \cos\beta a = 0 = \cos\dfrac{(2n-1)\pi}{2} \Rightarrow \beta_n = \dfrac{(2n-1)\pi}{2a}\; M(x) = B_1 \sin\dfrac{(2n-1)}{2a}x\; n = 1,2,3,\ldots\end{cases}$$

$$\beta = 0 \to M'' = 0 \Rightarrow M(x) = Ax + B$$

$$\left.\begin{array}{l} M(0)=0 \rightarrow B=0 \\ M'(a)=0 \rightarrow A=0 \end{array}\right\} \rightarrow M(x)=0 \Rightarrow$$ $\beta=0$ جواب نیست.

پس

$$-\frac{N''}{N}=-\wp^{2} \Rightarrow N''+\wp^{2}N=0 \Rightarrow N(y)=C_1 \sin\wp+C_2 \cos\wp y$$

$$N'=\wp\left(c_1 \cos\wp y-c_2 \sin\wp y\right)$$

$$N'(0)=0 \Rightarrow \wp c_1=0 \rightarrow \begin{cases} \wp=0 \\ c_1=0 \rightarrow c_2 \neq 0 \end{cases}$$

$$N(b)=0 \rightarrow c_2 \cos\wp b=0 \Rightarrow \cos\wp b=0=\cos\frac{(2m-1)\pi}{2} \Rightarrow \wp_m=\frac{(2m-1)\pi}{2b}\ m=1,2,3,\ldots$$

پس: $N(y)=C_2 \cos\frac{(2m-1)\pi}{2b} y$

$$\left.\begin{array}{l} N'=0 \Rightarrow A=0 \Rightarrow N(y)=B \\ N(b)=0 \rightarrow B=0 \end{array}\right\} \rightarrow N(y)=0 \Rightarrow$$ $\wp=0$ جواب نیست.

پس

پس:

$$U(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sin\frac{(2n-1)\pi}{2a}x.\cos\frac{(2m-1)\pi}{2b}y.\left(A_{nm}\cos c\lambda_{nm}t+B_{nm}\sin c\lambda_{nm}t\right)$$

$$\lambda_{nm}=\sqrt{\frac{(2n-1)^2\pi^2}{4a^2}+\frac{(2m-1)^2\pi^2}{4b^2}}$$

$$U(x,y,0)=0 \Rightarrow A_{nm}=0$$

$$U_t(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sin\frac{(2n-1)}{2a}x.\cos\frac{(2m-1)\pi}{2b}y\left(c\lambda_{nm}B_{nm}\cos c\lambda_{nm}t\right)$$

$$U(x,y,t)=xy(x+y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}c\lambda_{nm}B_{nm}\sin\frac{(2n-1)\pi}{2a}x\cos\frac{(2m-1)\pi}{2b}y$$

$$B_{nm}=\frac{4}{abc\lambda_{nm}}\int_0^b\int_0^a xy(x+y)\sin\frac{(2n-1)\pi}{2a}x\cos\frac{(2m-1)\pi}{2b}y$$

پاسخ 3 قسمت د:

چون شرایط مرزی دقیقا با شرایط مرزی قسمت الف برابر است پس مانند قسمت الف خواهیم داشت:

$$U(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sin\frac{n\pi}{a}x\sin\frac{m\pi}{b}y\left(A_{nm}\sin c\sqrt{\left(\frac{n\Pi}{a}\right)^2+\left(\frac{m\Pi}{b}\right)^2}\,t+B_{nm}\cos c\sqrt{\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2}\,t\right)$$

$$U(x,y.t)=e^{-y}\sin\frac{\pi x}{a}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}B_{nm}\sin\frac{n\pi}{a}x\sin\frac{m\pi}{b}y$$

$$B_{nm}=\frac{4}{ab}\int_0^b\int_0^a e^{-y}\sin\frac{\pi x}{a}\sin\frac{n\pi}{a}x\sin\frac{m\pi}{b}y$$

$$U_t=(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sin\frac{n\pi}{a}x\sin\frac{m\pi}{b}y\left(A_{nm}c\lambda\cos c\lambda t-B_{nm}c\lambda\sin c\lambda t\right)$$

$$U_t(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sin\frac{n\pi}{a}x\sin\frac{m\pi}{b}y\left(A_{nm}c\lambda\right)=4\cos\frac{5\pi}{b}y\sin\frac{\pi x}{2a}$$

$$A_{nm}=\frac{4}{abc\sqrt{\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2}}\int_0^b\int_0^a 4\cos\frac{5\pi}{b}y\sin\frac{n\pi}{2a}\sin\frac{n\pi}{a}x\sin\frac{m\pi}{b}y$$

---

4) $C^2U_{nn}=U_{tt}$ $\quad x>0 \quad ,x>0$

$U_n(0,t)=0 \quad ,U(x,0)=e^{-x2} \quad U_t(x,0)=0$

جواب: $\lim_{x\to\infty}|u(x,t)|<M$

$$F_c\{u(x,t)\}=u(w,t)=\int_0^{\infty}u(x,t)\cos wxdx$$

$$u(x,t)=\frac{2}{\Pi}\int_0^{\infty}u(w,t)\cos wxdw$$

$$F_c\{u_{tt}(x,t)\}=u_{tt}(w,t)$$

$$F_c\{u_{xx}\}=\int_0^{\infty}u_{xx}\cos wxdx \qquad \Rightarrow W=\cos\omega x\to dW=-\omega\sin\omega xdx$$

$$dV = u_{xx}dn \to V = u_x$$

$$I = u_x \cos\omega x\Big|_0^\infty + \omega\int_0^\infty u_x \sin\omega x dx$$

I_1

$$\begin{cases} W = \sin\omega x & \to dW = \varpi\cos\omega x dx \\ dV = u_x dx & \to V = u \end{cases}$$

$$I_1 = u\sin\omega x\Big|_0^\infty - \omega\int_0^\infty u\cos wxdx = -wu(w,t)$$

$$F_c\{u\} = -w^2 u(w,t)$$

$$-c^2w^2u(w,t) = u_{tt}(w,t) \to u_{tt}(w,t) + c^2w^2u(w,t) = 0$$

$$\to D^2 + c^2w^2 = 0 \to D = \pm icw$$

$$\Rightarrow u(w,t) = A\sin cwt + B\cos cwt \qquad ,u(w,0) = B, u_t(w,0) = Acw$$

$$u(w,0) = \int_0^\infty u\ \ (x,0)\cos wxdx = \int_0^\infty e^{-x2}\cos wxdx$$

$$u_t(w,0) = \int_0^\infty u_t(x,0)\cos wxdx = 0 \to A = 0$$

$$u(w,0) = \int_0^\infty e^{-x2}\left(\frac{e^{iwx} + e}{2}\right) \to F\{e^{-x2}\} = \int_{-\infty}^\infty e^{-x2}.e^{-iwx}dx = \int_{-\infty}^\infty e^{-x2}(\cos wx - i\sin wn)dx$$

$$= 2\int_0^\infty e^{-x2}\cos wxdx = \sqrt{\pi}e^{-\frac{w2}{4}}$$

$$u(w,t) = \frac{\sqrt{\pi}}{2}e^{\frac{-w^2}{4}}\cos cwt \to u(x,t) = \frac{1}{4\sqrt{\pi}}\int_0^\infty e^{\frac{-w^2}{4}}\cos cwt\cos wxd\omega$$

---

5) $\frac{5^2u}{5r^2} + \frac{2}{r}\frac{5u}{5r} = \frac{1}{c^2}\frac{5^2u}{5t^2}$ $\qquad a < r < b, t > 0$

$u(a,t) = 0 \qquad ,u(b,t) = 0 \qquad ,u(r,0) = e^{-r} \qquad ,u_t(r,0) = 0$

معادله همگن بودن و شرایط مرزی آن نیز همگن است. معادله در دستگاه کروی بوده و از θ است.

جواب: $u(r,t) = R(r)T(t) \qquad , \frac{R'' + \frac{2}{r}R'}{R} = \frac{T''}{c^2t} = \lambda$

$R''+\frac{2}{r}R'-\lambda R=0 \quad \rightarrow r^2R''+2rR'-\lambda r^2R=0 \leftarrow$ این معادله را نمی

شود حل کرد پس از روش دیگری استفاده می کنیم.

$$u_{rr}+\frac{2}{r}u_r=\frac{1}{c^2}u_{tt}$$

$$u_{rr}+\frac{2}{r}u_r=\frac{1}{r}\frac{\partial^2}{\partial r^2}(ru)=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$$\rightarrow \frac{\partial^2}{\partial r^2}(ru)=\frac{r}{c^2}\frac{\partial^2 u}{\partial t^2}\rightarrow\frac{\partial^2(ru)}{\partial r^2}=\frac{1}{c^2}\frac{\partial^2(ru)}{\partial t^2}\xrightarrow{\text{تغییر متغیر}}$$

$$V(r,t)=ru(r,t) \qquad ,V_{rr}=\frac{1}{c^2}V_{tt}$$

$$V(a,0)=au(a,t)=0 \qquad ,V(b,t)=bu(b,t)=0$$

$$V(r,0)=rF(r)=F(r) \qquad V_t(r,0)=ru_t(r,0)=rg(r)=G(r)$$

$$V_{rr}=\frac{1}{c}V_{tt}$$

از جداسازی متغیرها $\rightarrow V(r,t)=R(r)T(t)\rightarrow\frac{R''}{R}=\frac{T''}{C^2T}=-\beta^2$

$$\begin{cases} R''+\beta^2R=0\Rightarrow R=c_1\sin\beta r+c_2\cos\beta r \\ R(a)=0 \\ R(b)=0 \end{cases}$$

اگر $\left.\begin{array}{l} x=r-a \\ L=b-a \end{array}\right\}\Rightarrow V_r=\frac{\partial V}{\partial r}=\frac{\partial V}{\partial x}\cdot\frac{\partial x}{\partial r}$

$V_{xx}=\frac{1}{C^2}V_{tt} \qquad ,V_t(x,0)=(x+a)g(x+a)=G(x)$ ناقص است.

سوال 6 الف: $C\ \nabla^2u=u_{tt}$

روی تمام مرزها

$$\begin{cases} 0<x<a \\ 0<y<b \\ 0<z<d \\ t>0 \end{cases} \qquad \begin{cases} u=0 \\ u(x,y,z,0)=F(x,y,z) \\ u_t(x,y,z,0)=g(x,y,z) \end{cases}$$

$$C^2(u_{xx}+u_{yy}+u_{zz})=u$$
$$u(x,y,z,t)=M(x)N(y)Q(z)p(t)$$

$$C^2(M''NQP+MN''QP+MNQP'')=MNQP'' \qquad \frac{1}{C^2MNQP}$$ ضرب طرفين در

$$\frac{M''}{M}+\frac{N''}{N}+\frac{Q''}{Q}=\frac{P''}{PC^2}=\lambda \qquad (\lambda<0)$$

$$P''-\lambda c^2P=0 \Rightarrow P(t)=A_1\sin c\sqrt{\lambda}t+A_2\cos c\sqrt{\lambda}t$$
$$\frac{M''}{M}=\lambda-\left(\frac{N''}{N}+\frac{Q''}{Q}\right)=\mu \Rightarrow M''-\mu M=0 \Rightarrow N(x)=B_1\sin\sqrt{\mu}x+B_2\cos\sqrt{\mu}x$$
$$\frac{N''}{N}=\lambda-\left(\frac{M''}{M}+\frac{Q''}{Q}\right)=\alpha \Rightarrow N''-\alpha N=0 \Rightarrow N(Y)=D_1\sin\sqrt{\alpha}Y+D_2\cos\sqrt{\alpha}Y$$
$$\frac{Q''}{Q}=\lambda-\left(\frac{N''}{N}+\frac{M''}{M}\right)=\beta \Rightarrow Q''-\beta Q=0 \Rightarrow Q(z)=E_1\sin\sqrt{\beta}z+E_2\cos\sqrt{\beta}z$$

$$\lambda=\eta+\alpha+\beta \Rightarrow \sqrt{\lambda}=\sqrt{\mu+\alpha+\beta}$$

$$\begin{cases} u(0,y,z,t)=0 \Rightarrow B_2=0 \\ u(a,y,z,t)=0 \Rightarrow \sin\sqrt{\mu}a=0\sin m\pi \Rightarrow \sqrt{\mu}=\frac{m\pi}{a}m=1,2,3,... \end{cases}$$

$$M(x)=B_1\sin\frac{m\pi}{a}x$$

$$\begin{cases} u(x,0,z,t)=0 \Rightarrow D=0 \\ u(x,b,z,t)=0 \Rightarrow \sin\sqrt{\alpha}b=0=\sin n\pi \Rightarrow \sqrt{\alpha}=\frac{n\pi}{b}n=1,2,3,... \end{cases}$$

$$N(Y)=D_1\sin\frac{n\pi}{b}Y$$

$$\begin{cases} u(x,y,0,t)=0 \Rightarrow E=0 \\ u(x,y,d,t)=0 \Rightarrow \sin\sqrt{\beta}d=\sin k\pi=0 \end{cases}$$

$$\sqrt{\beta}=\frac{k\pi}{d}\, k=1,2,3,\ldots$$

$$q(z)=E_1\sin\frac{k\Pi}{d}z$$

$$u_{mnk}=B_1D_1E_1\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}Y\sin\frac{n\pi}{d}z\left(A_1\sin c\sqrt{\lambda}t+A_2\cos c\sqrt{\lambda}t\right)$$

$$u(x,y,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}z\left(A_{mnk}\cos c\sqrt{\lambda}t+B_{nmk}\sin c\sqrt{\lambda}t\right)$$

$$u(x,y,z,0)=F(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}A_{mnk}\sin\frac{n\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}z$$

$$A_{mnk}=\frac{8}{abd}\int_0^d\int_0^b\int_0^a F(x,y,z)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}zdxdydz$$

محاسبه λ

$$\sqrt{\lambda}=\sqrt{\alpha+\beta+\mu}=\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{k}{d}\right)^2}$$

$$u(x,y,z,0)=g(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}c\sqrt{\lambda}B_{nmk}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}z$$

$$B_{mnk}=\frac{8}{abdc\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{k}{d}\right)^2}}\int_0^d\int_0^b\int_0^a g(x,y,z)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}zdxdydz$$

سوال 6 قسمت (ب) :

$$C^2\nabla^2u=u_{tt}$$

$$\begin{cases} 0 < x < a \\ 0 < y < b \\ 0 < z < d \\ t > 0 \end{cases} \quad \begin{cases} u(0,y,z,t) = 0 \\ u_x(a,y,z,t) = 0 \end{cases} \quad \begin{cases} u(x,b,z,t) = 0 \\ u(x,0,z,t) = 0 \end{cases} \quad \begin{cases} u(x,y,0,t) = 0 \\ u(x,y,d,t) = 0 \end{cases}$$

$$\begin{cases} u(x,y,z,0) = ze^{x+y} \\ u_t(x,y,z,0) = 0 \end{cases}$$

$$C^2(u_{xx} + u_{yy} + u_{zz}) = u \qquad u(x,y,z,t) = M(x)N(y)Q(z)P(t)$$

$$C^2(M''NQP + MN''QP + MNQ''P) = P''MNQ$$ ضرب در طرفین $\frac{1}{C^2 MNQP}$

$$\frac{M''}{M} + \frac{N''}{N} + \frac{Q''}{Q} = \frac{P''}{C^2 P} = \lambda(\lambda < 0)$$

$$P'' - C^2\lambda P = 0 \Rightarrow P(t) = A_1 \sin c\sqrt{\lambda}T + A_2 \cos C\sqrt{\lambda}T$$

$$\frac{M''}{M} = \lambda - \left(\frac{N''}{N} + \frac{Q''}{Q}\right) = \mu \Rightarrow M(x) = B_1 \sin\sqrt{\mu}x + B_2 \cos\sqrt{\mu}x$$

$$\frac{N''}{N} = \lambda - \left(\frac{M''}{M} + \frac{Q''}{Q}\right) = \alpha \Rightarrow N(y) = D_1 \sin\sqrt{\alpha}Y + D_2 \cos\sqrt{\alpha Y}$$

$$\frac{Q''}{Q} = \lambda - \left(\frac{M''}{M} + \frac{N''}{N}\right) = B \Rightarrow Q(z) = E_1 \sin\sqrt{\beta}Z + E_2 \cos\sqrt{\beta}Z$$

$$\begin{cases} u(0,y,z,t) = 0 \Rightarrow M(0) = 0 \Rightarrow B_2 = 0 \\ u(a,y,z,t) = 0 \Rightarrow M'(a) = 0 \Rightarrow M'(x) = B\sqrt{\mu}\cos\sqrt{\mu}x \end{cases} \qquad M(x) = B_1 \sin\sqrt{\mu}x$$

چون ق ق غ $$\sqrt{\mu} = 0 \Rightarrow M'' = 0 \Rightarrow \begin{cases} M(x) = F_1X + F_2 \\ M(x) = 0 \Rightarrow F_2 = 0 \\ M'(a) = 0 \Rightarrow F_1 = 0 \Rightarrow M(a) = 0 \end{cases}$$

داریم.

$$\cos\sqrt{\mu}a = \cos\left(\frac{2m-1}{2}\right)\pi \Rightarrow \sqrt{\mu} = \frac{(2m-1)\pi}{2a}$$

$m = 1,2,3,\ldots$

در نتیجه

داریم:

$$M(x) = B_1 \sin\left(\frac{2m-1}{2a}\right)\pi$$

$$\begin{cases} u_y(x,0,z,t)=0 \\ u(x,b,z,t)=0 \end{cases}$$

$$\begin{cases} N'(y)=\sqrt{\alpha}(D_1\sin\sqrt{\alpha}y-D_2\sin\sqrt{\alpha}y) \\ N'(0)=0\Rightarrow\sqrt{\alpha}D_1=0\begin{cases}\sqrt{\alpha}=0\Rightarrow\begin{cases}N(y)=f_1y+f_2\\ N(b)=0\to f_2=0\\ N'(0)=0\to f_1=0\end{cases} \\ D_1=0\end{cases}\end{cases}$$

غ ق ق

قابل قبول

$$\begin{cases} N(y)=D_2\cos\sqrt{\alpha}y \\ N(b)=0\Rightarrow\sqrt{\alpha}b=\dfrac{(2n-1)\pi}{2}\Rightarrow\sqrt{\alpha}=\dfrac{(2n-1)\pi}{2b} \end{cases}$$

$n=1,2,3,\ldots$

$$N(y)=D_2\cos\left(\frac{2n-1}{2b}\right)\pi y$$

$$\begin{cases} u(x,y,0,t)=0 \\ u(x,y,d,t)=0 \end{cases}\qquad \begin{matrix}\Rightarrow E_2=0\\ \sin\sqrt{\beta}d=\sin k\pi\end{matrix}\qquad \sqrt{\beta}=\frac{k\pi}{d}$$

$k=1,2,\ldots$

$$q(z)=E_1\sin\frac{k\pi}{d}z$$

$$\lambda=\alpha+\beta+\mu\Rightarrow\sqrt{\lambda}=\sqrt{\left(\frac{k\pi}{d}\right)^2+\left(\frac{2n-1}{2b}\pi\right)^2+\left(\frac{2m-1}{2a}\pi\right)^2}$$

$$u(x,y,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\sin\frac{(2m-1)}{2a}\pi x\right)\left(\cos\frac{(2n-1)}{2b}\pi\right)\left(\sin\frac{k\pi}{d}z\right)\left(A_{mnk}\cos c\sqrt{\lambda}t+B_k mnk\sin c\sqrt{\lambda}t\right)$$

$$u_t(x,y,z,0)=0\to B_{nmk}=0$$

$$u(x,y,z,0)=F(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}A_{mnk}\sin\frac{2m-1}{2a}\pi x\cos\frac{2n-1}{2b}\pi y\sin\frac{k\pi}{d}z$$

$$A_{mnk}=\frac{8}{abd}\int_0^b\int_0^b\int_0^a F(x,y,z)\sin\left(\frac{2m-1}{2a}\pi x\right)\cos\left(\frac{2n-1}{2b}\pi y\right)\sin\frac{k\pi}{d}z\,dxdydz$$

$$F(z,y,z)=ze^{x+y}$$

$$A_{nmk}=\frac{32d\left(\pi-2m\Pi+2e^a a(-1)^m\right)\left(e^b\pi(-1)^n-2b-2e^b n\pi(-1)^n\right)(-1)^k}{\left(4a^2+4m^2\pi^2-4m\pi^2+\pi^2\right)\left(4b^2+4m^2\pi^2-4n\pi^2+\pi^2\right)k\pi}$$

سوال 6 ج

با توجه به سوال 6 قسمت الف داریم:

$$u(x,y,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}z\left(A_{mnk}\cos c\sqrt{\lambda}t+B\sin c\sqrt{\lambda}t\right)$$

$$A_{mnk}=\frac{8}{abd}\int_0^d\int_0^b\int_0^a F(x,y,z)\sin\frac{m\pi}{a}x.\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}zdxdydz$$

$$u(x,y,z,0)=\sin x.\cos y.\cosh z$$

$$A=\frac{8}{abd}.\frac{a(m\pi\sin a(-1)^m)}{a^2-m^2\pi^2}.\frac{b(n\pi\sin b(-1)^n)}{b^2-n^2\pi^2}.\frac{d(2k\pi e^d-(-1)^k k\pi e^{2d}-k\pi(-1)^k}{2(d^2+k^2\pi^2)}$$

$$B_{mnk}=\frac{8}{abdc\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{k}{d}\right)^2}}\int_0^d\int_0^b\int_0^a d(x,y,z)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}zdxdydz$$

$$g(x,y,z)=0\Rightarrow B=0$$

$$\sqrt{\lambda}=\sqrt{\alpha+\beta+\mu}=\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{k}{d}\right)^2}$$

سوال 6 قسمت د)

با توجه به قسمت الف سوال 6 داریم:

$$u(x,y,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}z(A_{mnk}\cos c\sqrt{\lambda}t+B\sin c\sqrt{\lambda}t)$$

$$A_{mnk}=\frac{8}{abd}\int_0^d\int_0^b\int_0^a F(x,y,z)\sin\frac{m\pi}{a}x.\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}zdxdydz$$

$$F(x,y,z)=u(x,y,z,0)=0\rightarrow A_{mnk}=0$$

$$B_{mnk}=\frac{8}{abdc\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{k}{d}\right)^2}}\int_0^d\int_0^b\int_0^a g(x,y,z)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sin\frac{k\pi}{d}zdxdydz$$

$$g(x,y,z)=u(x,y,z,0)=z^2\sin x.e^{-y}$$

$$B_{mnk}=\frac{8}{abdc\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{k}{d}\right)^2}}.\frac{+a(m\pi\sin a(-1)^x)}{a^2-m^2\pi^2}.\frac{d^3(2(-1)^k-2-k^2\pi^2(-1)^k)}{k^3\pi^3}$$

$$.\frac{b\left(n\pi-e^b n\pi(-1)^n\right)}{b^2+n^2\pi^2}$$

$$B_{mnk}=\frac{8m\sin a(-1)^m n(1-e^b(-1)^n)(2(-1)^k-2-k^2\pi^2(-1)^k)d^2}{c(a^2-m^2\pi^2)(b^2+n^2\pi^2)k^3\pi^2}$$

سوال 7)