$U_{xx} = U_t \qquad 0 < x < 1 \qquad , t > 0$

$U(0,t) = u(l,t) = u_0$ غیر همگن

$W(n,t) + r(x) = u(n,t)$

$\alpha(W_{xx} + r_{xx}) = Wt$

$\alpha r_{xx} = 0 \to r = A \to r(x) = A_n + B \begin{cases} r(0) = u \to B = u_0 \\ r(l) = u \to A = 0 \end{cases}$

$u(n,t) = W(n,t) + u_0 \qquad r(x) = -u_0$

$\alpha W_{xx} = W_t \qquad W(0,t) = W(l,t) = 0 \qquad W(n,0) = F(x) - u_0$

$W(x,t) = M_{(x)} nt \qquad \frac{M''}{M} = \frac{N''}{\alpha N} = -\beta^2$

$M = A_1 \varepsilon . \frac{n\pi}{l} x, N(t) = Be^{-\alpha\beta^2 t}, \beta_n = n\pi / l$

$W(x,t) = \sum_{n=1}^{\infty} Ae^{-\alpha\beta^2 t} \varepsilon . \frac{n\pi}{l} x \qquad W(x,0) = \sum_{n=1}^{\infty} A_n \varepsilon . \frac{n\pi}{l} n$

$u(x,t) = w(x,t) + u_0 \qquad A_n = 2/l \int_0^l (4x^2 - 1) \varepsilon . \frac{n\pi}{l} xdx$

ب) $r_{xx} = 0 \to r(x) = Ax + B \qquad r_{(0)} = u_1 \to B = u_1 \qquad r_{(l)} = u_2 \to A = \frac{u_2 - u_1}{l}$

$r_{(x)} = \left(\frac{u_2 - u_1}{l}\right) x + u_1 \qquad W_{xx} = w_t$

$W(0,t) = W(l,t) = 0 \qquad W(x,0) = F(x) - \frac{u_2 - u_1}{l} x - u_1$

$W(x,t) = \sum_{n=1}^{\infty} Ae\varepsilon . \frac{n\pi}{l} x \qquad A_n = 2/l \int_0^l \varepsilon . \pi/l \, n\varepsilon . \frac{n\Pi}{l} xdx$

$u(x,t) = W(x,t) + r(x) \qquad W(x,0) = \sum_{n=1}^{\infty} A_n \varepsilon . \frac{n\pi}{l} n \uparrow$

ج)

$N < 0 : N = -\beta$

$M(x) = A_1 \varepsilon . \beta \to M(x) = A_1 \beta_1 \cos\beta \xrightarrow{A_1 \beta \neq 0} \beta = \frac{2n-1}{2l} \pi, n = 1,2,...$

$N(t) = Be^{-\alpha\beta^2 t} \qquad u = \sum_{n=1}^{\infty} A_n e^{-\alpha\left(\frac{2n-1}{2l}\pi\right)^2 t} . \varepsilon . \frac{2n-1}{2l} \pi x$

$$u(x,0)=\cosh\beta_x=\sum_{n=1}^{\infty}A_n\varepsilon.\frac{2n-1}{2l}\pi x \qquad \text{F.S.S}$$

$$A_n={}^{2}\!/\!_{l}\int_0^l \cosh\beta x.\varepsilon.\frac{2n-1}{2l}\pi x\,dx$$

د) $I<0$

$$M(x)=A_2\cos\left(\frac{2n-1}{2l}\pi\right)x,\ Nt=Ae^{-\alpha\beta^2 t}$$

$$u(x,t)=\sum_{n=1}^{\infty}A_n e^{-\alpha\left(\frac{2n-1}{2l}\right)^2 t}\cos\left(\frac{2n-1}{2l}\pi\right)x \qquad \text{F.C.S}$$

$$A_n=\frac{2}{l}\int_0^l {}^{x^3}\!/\!_{3}\cos\left(\frac{2n-1}{2l}\pi\right)x\,dx$$

هـ) $r(x)=-\frac{u_1}{l}x+u_1 \qquad W(x,0)=\varepsilon\beta x-u_1+\frac{u_1}{l}x$

$$\rightarrow u(x,t)-u_1+\frac{u_1}{l}x=\sum_{n=1}^{\infty}A_n e^{-\alpha\left(\frac{n\pi}{l}\right)^2 t}\varepsilon.\frac{n\pi}{l}x$$

$$A_n=\frac{2}{l}\int_0^l\left[\varepsilon.\beta-u\left(1-{}^{x}\!/\!_{l}\right)\right]\varepsilon.\frac{n\pi}{l}x\,dx$$

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9) $r_{(x)}=u_0x$

$$W(x,0)=2n-1-u_0x=(2-u_0)x-1\rightarrow u(x,t)=u_0x=\sum_{n=1}^{\infty}A_n e^{-\alpha\beta^2 t}\varepsilon.\frac{n\pi}{l}x$$

$$A_n=\frac{2}{l}\int_0^l\left[(2-u)x-1\right]\varepsilon.\frac{n\pi}{l}x$$

سوال 8 قسمت الف :

$\alpha u_{xx}-A_u=u_t \qquad 0<x<l \qquad t>0 \qquad u(0,t)=u(l,t)=0 \qquad u(x,0)=0$

$u(x,t)=M(z)N(t)$ $\qquad u_{xx}=MN''$ $\qquad ut=N'M$

$\alpha M''N-AMN=N'M$ $\xrightarrow{\text{تقسیم بر } MN}$

$$\frac{\alpha M''}{N}-A=\frac{N'}{N}\rightarrow\frac{M''}{M}-\frac{A}{\alpha}=\frac{N'}{\alpha N}$$

تابع w در جهت x تناوب است پس:

$$\frac{M''}{M}=-r''^2\rightarrow M''+r^2M=0\rightarrow M(x)=A_1\sin rx+Ar\cos rx$$

$M(0)=0\rightarrow M(0)=A_2+A_2=0$ $\qquad M(x)=A_1\sin r\alpha$

$M(l)=0\rightarrow A_1\sin\wp l=0$ $\qquad \wp l=n\pi\rightarrow\wp=\frac{n\pi}{l}\rightarrow M(x)=A_1\sin\frac{n\pi}{l}x$

$\frac{N'}{N}=-\beta^2\alpha\rightarrow\frac{N'}{N}+\beta^2\alpha=0$ $\qquad \frac{dN}{dT}=-\beta^2Nd\Rightarrow\frac{dN}{N}=-\beta^2\alpha dt\rightarrow lN=-\beta^2\alpha t$

$$N=N.e^{-\beta^2\alpha t}\rightarrow N(t)=N.e\rightarrow N(t)=N.e^{-\beta^2\alpha t}$$

با توجه به روابط بالا داریم: $-\wp^2-\frac{A}{\alpha}=-\beta^2$

$$\wp^2+A/\alpha=\beta^2\rightarrow\frac{n^2\pi^2}{l^2}+\frac{A}{\alpha}=\beta^2$$

$$u(x,t)=\sum_{n=1}^{\infty}A\sin\frac{n\pi}{l}xe^{-\left((\frac{n\pi}{l})^2+\frac{A}{\alpha}\right)\alpha t}\rightarrow u(x,0)=\sum_{n=1}^{\infty}An\sin\frac{n\pi}{l}x=0$$

$A_n=0$ $\qquad u(x,t)=0$

قسمت دوم ب)

$u(x,t)=M(x)N(t)$ $\qquad u(0,t)=u_x(l,t)=0$

$\alpha\frac{M''}{M}-A=\frac{N'}{N}=-\beta^2$ $\qquad u(x,0)=x^2+1$

به علت اینکه تابع w در جهت x متناوب است پس:

$$\frac{M''}{M}-\frac{A}{\alpha}=\frac{N'}{\alpha N}\rightarrow\frac{M''}{M}=-\wp^2$$

$\frac{M''}{M}=-\wp^2\rightarrow M(x)=A_1\sin rx+A\cos\wp x$ $\qquad M(0)=0\rightarrow A_2=0$

$$M(\alpha) = A_1 \sin \wp x \rightarrow M'(x) = A_1 r \cos \wp x \Rightarrow M(l) = 0 \qquad \wp l = \frac{(2n-1)\pi}{2} \rightarrow \wp = \frac{(2n-1)\pi}{2l}$$

$$-\wp^2 + \beta^2 = \frac{A}{\alpha} \rightarrow \beta^2 - \wp^2 = \frac{A}{\alpha} \rightarrow \frac{N'}{N\alpha} = -\beta^2 \rightarrow N = N.e^{-\alpha\beta^2 t}$$

$$N(t) = n.e^{-\alpha\beta^2 t}$$

$$\beta^2 = \frac{A}{\alpha} + \frac{(2n-1)^2 \pi^2}{4l^2} \rightarrow N(t) = N.e^{-\alpha\left(\frac{A}{\alpha} + \frac{(2n-1)^2\pi^2}{4l^2}\right)t}$$

$$u(x,0) = F(x) = \sum_{n=1}^{\infty} A_n \sin \frac{(2n-1)\pi}{2l} x \qquad F(x) = x^2 + 1$$

$$A_n = 2/l \int_0^l F(x) \sin \frac{(2n-1)}{2l} x dx = 2/l \int_0^l (x^2+1) \sin\left(\frac{2n-1\pi}{2l}\right) x dx$$

α

x^2+1	+	$\sin \alpha x$
$2x$		$-\frac{1}{\alpha}\cos dx$
2		$-\frac{1}{\alpha^2}\sin \alpha x$
0		$\frac{1}{3}\cos \alpha x$

$$-\frac{x^2+1}{\alpha}\cos dx + \frac{2x}{\alpha^2}\sin xd + \frac{2}{\alpha^3}\cos dx \Big|_0^l$$

$$-\frac{l^2+1}{\alpha}\cos \alpha l + \frac{2l}{\alpha^2}\sin \alpha l + \frac{2}{\alpha^3}\cos \alpha l - \left[-\frac{1}{\alpha} + \frac{2}{\alpha^3}\right]$$

$$-\frac{l^2+1}{\frac{(2n-1)}{2l}}\cos\frac{(2n-1)\pi}{2} + \frac{2l}{\frac{(2n-1)^2\pi^2}{4l^2}}\sin\frac{(2n-1)\pi}{2}$$

$$+\frac{2}{\frac{(2n-1)^3\pi^3}{8l^3}}\cos\frac{(2n-1)\pi}{2} + \frac{2l}{(2n-1)\pi} - \frac{2}{\frac{(2n-1)^3\pi^3}{8l^3}}$$

$$= \frac{2l}{\frac{(2n-1)^2\pi^2}{4l^2}}(-1)^n + \frac{2l}{(2n-1)\pi} - \frac{2}{\frac{(2n-1)^3\pi^3}{8l^3}}$$

8) $\alpha u_{xx} - Au = u_t \qquad 0 < x < l \qquad t > 0$

ج) $u_x(0,t) = 0, u(l,t) = 0, u(x,0) = 0$

$$u(x,t) = F(x)G\pi(t) \Rightarrow aF''G - AFG = FG' \Rightarrow \frac{F''}{F} - \frac{A}{\alpha} = \frac{g'}{\alpha G} \Rightarrow \frac{F''}{F} = \frac{G'}{\alpha G} + \frac{A}{\alpha} = -P^2$$

$$\Rightarrow F'' + P^2 F = 0 \Rightarrow F(x) = B_1 \sin Px + B_2 \cos Px \Rightarrow F'(x) = B_1 P \cos Px - B_2 p \sin Px$$

$$F'(0) = 0 \Rightarrow B_1 p = 0 \Rightarrow \begin{cases} B_1 = 0 \\ p = 0 \end{cases} \longrightarrow F(x) = BCospx$$

$$F(l)=0 \Rightarrow Cospl=0=Cos\frac{2n-1}{2}\pi \Rightarrow p=\frac{(2n-1)\pi}{2l} \Rightarrow$$

$$G'+\alpha\left(\frac{A}{\alpha}+p^2\right)G=0 \Rightarrow G=C_n e^{-(A+\alpha p^2)t}$$

$$p=0 \Rightarrow F''=0 \Rightarrow F(x)=O_1x+D_2 \qquad F'(0)=0 \Rightarrow D_1=0, F(l)=0 \Rightarrow D_2=0 \Rightarrow F(x)=0$$

$$\Rightarrow u(x,t)=\sum_{n=1}^{\infty} C_n Cos\frac{(2n-1)\pi}{2l} x e^{-(A+\alpha p^2)t}$$

$$u(x,0)=0 \Rightarrow 0=\sum_{n=1}^{\infty} C_n Cos\frac{(2n-1)\pi}{2l} x \Rightarrow C_n=0$$

$$\Rightarrow u(x,t)=0$$

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8) (د)

$$u(0,t)=0, u(l,t)=0, u(x,0)=\sin x$$

$$F(x)=B_1\sin px+B_2\cos px \Rightarrow F'(x)=B_1p\cos px-B_2p\sin px$$

$$F'(0)=0 \Rightarrow B_1p=0 \rightarrow \begin{cases} B_1=0 \\ p=0 \end{cases} \longrightarrow F'(x)=-B_2p\sin px$$

$$F'(l)=0 \qquad B_2\neq 0 \Rightarrow \begin{cases} p=0 \\ \sin pl=0 \end{cases} \longrightarrow \sin pl=\sin n\pi \Rightarrow p_n=\frac{\pi}{nl}$$

$$\Rightarrow F(x)=B_2\cos\frac{n\pi}{l}x$$

$$G(t)=c_n e^{-(A+\alpha p^2)t} \quad \Rightarrow u(x,t)=\sum_{n=1}^{\infty} c_n Cos\frac{n\pi}{l} x e^{-(A+\alpha p^2)t}$$

$$u(x,0)=\sin x \Rightarrow \sin x=\sum_{n=1}^{\infty} c_n Cos\frac{n\pi}{l}x$$

$$\Rightarrow C_n=\frac{1}{2l}\int_0^l \sin x Cos\frac{n\pi}{l}x dx$$

جواب سوال 9-

$$\nabla^2 u=u_t \qquad 0<x<\pi \qquad ,0<y<\pi \qquad ,t>0$$

الف) $u=0$ روی تمام مرزها $\qquad u(x,y,0)=x(\pi-y)^2+\sin x\cos y$

-روش جداسازی متغیرها:

$u=\phi(x,y)T(t)$

$\{\nabla^2\phi+\lambda\phi=0 \qquad 0<x<\pi \qquad 0<y<\pi$

ϕروی مرز صفر است.

$T(t)=e^{-(x^2+m^2)t}$

می توان نوشت:

$\phi(x,y)=M(x).N(y)$

$\phi_{mn}=\sin nx.\sin my \qquad n,m=1,2,...$

$$u(x,y,t)=\sum_{n=1}\sum_{m=1}a_{nm}(t)\sin nx\sin my$$

$a_{nm}(t)=C_{nm}e^{-(n^2+m^2)t}$

وقتی: $t=0\rightarrow a_{nm}=c_{nm}$

$$u=\sum_{n=1}\sum_{m=1}c\sin nx\sin my=x(\pi-y)^2+\sin x\cos y$$

$$C_{nm}=\frac{2}{\pi}\times\frac{2}{\pi}\int_0^\pi\int_0^\pi\left[x(\pi-y)^2+\sin x\cos y\right] \qquad \sin nx\sin my dxdy$$

$$C\circ\circ=\frac{1}{\pi}\times\frac{1}{\pi}\int_0^\pi\int_0^\pi\left(x(x-y)^2+\sin x\cos y\right)dxdy$$

$$u(x,y,t)=c\circ\circ+\sum_{n=1}\sum_{m=1}c_{nm}\sin nx\sin mye^{-(x^2+m^2)t}$$

ب) $\begin{cases}u(0,y,t)=0\\ u(x,0,t)=0\end{cases} \qquad \begin{cases}u_x(\pi,y,t)=0\\ u(x,\pi,t)=0\end{cases} \qquad u(x,y,0)=\sinh x\cosh y$

$u_n(\pi, y, t) = 0$

$M'(x) = A_1\alpha\cos\alpha n - A_2\alpha\sin\alpha x$

$M'(\pi) = 0 \to A_1 = 0$

$M(0) = 0 \to A_2\cos y = A_2 \to A_2 \to M(x) = A_2\cos nx$

$\phi = \cos nx \sin my$

$T = e^{\left(n^2+m^2\right)t}$

$$u(x, y, t) = \sum_{m=1}\sum_{n=1} a_{nm}(t)\cos xn \sin mye^{-(m^2+n^2)t}$$

$t = 0 \to a_{mn} = c_{nm}$

$$u(x, y, t) = c\circ\circ + \sum_{n=1}\sum_{m=1} c_{mn} Cosxn \sin mye^{-(n^2+m^2)t}$$

$$C\circ\circ = \frac{1}{\pi^2}\int_0^\pi\int_0^\pi(\sinh x\cosh y)dxdy$$

$$C_{mn} = \frac{4}{\pi^2}\int_0^\pi\int_0^\pi(\cosh y\sinh x)\sin my\cos nxdxdy$$

ج)

$u_x(0, y, t) = u(\pi, y, t) = 0$

$u_y(x, 0, t) = 0, u(x, \pi, t) = 0$

$u(x, y, 0) = 4x^2y^3$

$M'(0) = 0 \to A_1 = 0$

$$M'(\pi) = 0 \to -A_2\alpha\sin\alpha\pi = \sin M\pi \begin{cases}\alpha = 0 \\ \alpha\pi = M\pi \Rightarrow \alpha_m = N\end{cases}$$

$T(t) = e^{-(n^2+m^2)t}$

$N'(y) = B_1\beta Cos\beta y - B_2\beta Sin\beta y$

$\to B_1 = 0$

$$N(\pi) = B_2 Cos\beta\pi = 0 \begin{cases}B_2 \neq 0 \\ \beta_n = \left(\dfrac{2n-1}{2}\right)\end{cases}$$

$$u(x, y, t) = c\circ\circ + \sum_{n=1}\sum_{m=1} cCosmx\sin\left(\frac{2n-1}{2}\right)y$$

$$C\circ\circ = \frac{1}{\pi}\times\frac{1}{\pi}\int_0^\pi\int_0^\pi 4x^2y^3dxdy$$

$$C_{nm} = \frac{4}{\pi}\int_0^\pi\int_0^\pi 4x^2y^3\cos mn\sin\left(\frac{2n-1}{2}\right)ydxdy$$

د)

$$\begin{cases} u_x(0,y,t)=0 & ,u(\pi,y,t)=0 \\ u(x,0,t)=0 & ,u_y(x,\pi,t)=0 \end{cases}$$

$$M(x)=A_1\alpha Cos\alpha n - A_2\alpha\sin\alpha n$$

$$M'(0)=0 \to A_1=0$$

$$M(\pi)=0 \to A_2\cos\alpha\pi=0 \begin{cases} A_2\neq 0 \\ \alpha_\pi=\left(\frac{2n-1}{2}\right)\pi \Rightarrow \alpha_n=\left(\frac{2n-1}{2}\right) \end{cases}$$

$$M(x)=A_2\cos\left(\frac{2n-1}{2}\right)n$$

$$N(y)=B_1\sin\beta y+B_2\cos\beta y$$

$$N'(y)=B_1\beta\cos\beta y-\beta B_2\sin\beta y$$

$$N'(\pi)=0 \to B_1\beta\cos\beta\pi=0 \to \begin{cases} \beta=0 \\ \cos\beta\pi=0 \end{cases}$$

$$N(0)=0 \to \beta_2=0 \qquad \beta_n=\left(\frac{2n-1}{2}\right)$$

$$N(y)=B_1\sin\left(\frac{2n-1}{2}\right)y$$

$$u(n,y,t)=C\circ\circ+\sum_{n=1}\sum_{m=1}c\left(e^n\sin y\right)\sin\left(\frac{2n-1}{2}\right)y\cos\left(\frac{2n-1}{2}\right)n$$

$$C\circ\circ=\frac{1}{\pi^2}\int_0^\pi\int_0^\pi e^n\sin y\,dxdy$$

$$C_{nm}=\frac{4}{\pi^2}\int_0^\pi\int_0^\pi e^x\sin y\sin\left(\frac{2n-1}{2}\right)y\cos\left(\frac{2n-1}{2}\right)xdxdy$$

هـ)

$$\begin{cases} u_x(0,y,t)=u_x(\pi,y,t)=0 \\ u_y(x,0,t)=u_y(x,\pi,t)=0 \end{cases} \qquad u(x,y,0)=x+y$$

$M(x) = A_1 \sin \alpha n + A_2 \cos \alpha n$

$M'(x) = A_2 \alpha \sin \alpha x + A_1 \alpha \cos \alpha x$

$M(x) = \cos nx$

$M(y) = B_1 \sin \beta y + B_2 \cos \beta y$

$M'(y) = B_1 \beta \cos \beta y - B_2 \beta \sin \beta y$

$N'(0) = \rightarrow B_1 \beta \cos 0 = 0 \rightarrow \beta = 0$

$N'(\pi) = 0 \rightarrow B_1 \beta \cos \beta\pi = 0 \Rightarrow \cos \beta\pi = \left(\frac{2n-1}{2}\right)$

$N'(\pi) = 0 \rightarrow B_1 \beta \cos \beta\pi = 0 \Rightarrow \cos \beta\pi = \left(\frac{2n-1}{2}\right)\pi$

$$\beta_n = \left(\frac{2n-1}{2}\right)$$

$N(y) = B_1 \sin\left(\frac{2n-1}{2}\right)y$

$u(x,y,t) = C_{\circ\circ} + \sum_{n=1}\sum_{m=1} \sin\left(\frac{2n-1}{2}\right) y \cos nx e^{-(n^2+m^2)t}$

$C_{\circ\circ} = \frac{1}{\pi}\int_0^\pi\int_0^\pi (x+y)dxdy$

$C_{nm} = \frac{4}{\pi}\int_0^\pi\int_0^\pi (x+y)\cos xn \sin\left(\frac{2n-1}{2}\right)ydxdy$

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10) $\alpha u_{xx} = u_t \qquad x > 0, t > 0$

الف) $u(0,t) = 0 \qquad u(x,0) = x^2 e^{-x^2}$

تبدیل سینوسی نسبت به $x \rightarrow u(w,t) = F_s[u(x,t)]$

$F_s[u_{xx}] = u_x \sin wx\Big|_0^\infty - \omega\int_0^\infty u_x \cos \omega x dx = -\omega\left(u \cos \omega x\Big|_0^\infty + \omega\int u \sin \omega x dx\right) = -\omega^2 u(\omega,t)$

$$-\alpha\omega^2 u(\omega,t) = u_t(\omega,t)$$

$u(\omega,t) = A_1 e^{-\alpha\omega^2 t} \qquad F_s[u(x,0)] = \int_0^\infty x^2 e^{-x^2} \sin \omega x dx =$

$$\frac{1}{2i}\int_{2}^{\infty} x^2\left[e^{-(x-i\omega/2)^2-\frac{w^2}{4}}-e^{-(x+i\omega/2)^2-w^2/4}\right]dx$$

$$e^{-\omega^2/4}\Big/2i\int_{0}^{\infty} x^2\left[e^{-(x-i\omega/2)^2}-e^{-(x+i\omega/2)^2}\right]dx=\frac{e^{-\omega/4^2}}{2i}\left\{\int_{-i\omega/2}^{\infty}\left(z+\frac{iw}{2}\right)edz+\int_{i\omega/2}^{\infty}\left(z-i\omega/2\right)^2 e^{-z^2}\right\}$$

$$\frac{e^{-\omega^2/4}}{2i}\left\{\int_{-i\omega/2}^{\infty} z^2e^2dz+iw\int_{-iw/2}^{\infty} ze^{-z^2}dz+\int_{iw/2}^{\infty} t^2e^{-t^2}-i\omega\int_{i\omega/2}^{\infty} te^{-t^2}dt-\frac{\omega^2}{4}\int_{i\omega/2}^{\infty} e^{-t^2}dt\right\}$$

$$\frac{e^{-\omega/4^2}}{2i}\left\{-\frac{1}{2}ze^{-z^2}+\left[-\frac{w^2}{8}\int_{-iw/2}^{\infty} e^{-z^2}dz+i\omega\left(-1/2\right)e^{-z^2}\Big|_{-i\omega/2}^{\infty}+\left(-1/2\right)te^{-t^2}+\frac{i\omega}{2}e^{t^2}\Big|_{i\omega/2}^{\infty}+\left(\frac{1}{2}-\frac{\omega^2}{4}\right)\int_{i\omega/2}^{\infty} et^2dt\right.\right\}$$

$$\frac{e^{-\frac{w^2}{4}}}{2i}\left\{-1/2\frac{iw}{2}e^{w/4^2}-\frac{1}{2}iwe^{w/4^2}-\frac{1}{2}iwe^{w/4^2}+iw/2\,e^{w/4^2}+\left(\frac{1}{2}-\frac{\omega^2}{4}\right)\int_{0}^{\infty} e^{-(x-iw/2)^2}dx\right.$$

$$+\left(\frac{1}{2}-\frac{w}{4}\right)\int_{4}^{\infty} edx$$

$$I=\left(\frac{1}{2}-\frac{w^2}{4}\right)e^{w/4^2}\int_{0}^{\infty}\left[e^{-x^2+iwx}+e^{-x^2+iwx}\right]dx=e^{-*(x+\frac{iw}{2})^2}dx\int_{0}^{\infty} e^{-x^2}\left(e^{iwx}+e^{-iwx}\right)dx=\left(1-\frac{w^2}{2}\right)e^{w/4^2}\int_{0}^{\infty} e^{-x^2}\cos xdx$$

$$=\left(1-\frac{w^2}{2}\right)e^{w/4^2}\left(\sqrt{\pi/2}\,e^{-w/4^2}\right)$$

$u(w,0)=A_1=F_s\left[u(x,0)\right]=F(w)$ $\qquad$ $u(w,t)=F(w)e^{-\alpha w^2 t}$

$$u(x,t)=\frac{2}{\pi}\int_{0}^{\infty} F(x)e^{-\alpha w^2 t}\sin wxdw$$

ب) $u_x(0,t)=0$ $\qquad$ $u(x,0)xe^{-x^2}$

تبدیل کسینوسی نسبت به x

$$u(w,t)=F_c\left[u(x,t)\right]$$