$$F_c(u_{xx}) = u_x \cos wx\Big|_0^{\infty} + w\int u_x \sin wxdx = w\left[u\sin wx\Big|_0^{\infty} - w\int_0^{\infty} u\cos wxdx\right] = -w^2 u$$

$$-\alpha w^2 u = u_t \to u(w,t) = Ae^{-\alpha w^2 t}$$

$$u(w,0) = A = F_c\left[xe^{-x^2}\right] = \int_0^{\infty} xe^{-x^2}\cos wxdx = -\frac{1}{2}e^{-x^2}\cos wx\Big|_0^{\infty} - \frac{1}{2}w\int_0^{\infty} e^{-x^2}\sin wxdx$$

$$I = \int_0^{\infty} e^{-x^2}\sin wxdx \to \frac{\delta I}{\delta w} = \int_0^{\infty} xe^{-x^2}\cos wxdx = -\frac{1}{2}e^{-x^2}\cos wx\Big|_0^{\infty} - \frac{1}{2}wI$$

$$\frac{\delta I}{\delta w} = \frac{1}{2} - \frac{1}{2}wI \to I = e^{\int w/2\,dw}\left[\frac{1}{2}e^{\int w/2\,dw}dw + c\right] = e^{-w^2/4}\left[\frac{1}{2}e^{w^2/4} + c\right] \quad w=0 \to I=0 \to$$

$$\left[\frac{1}{2}+c\right] = 0 \to c = -1/2 \to I = \left(-e^{-w^2/4}+1\right)1/2$$

$$u(w,0) = 1/2 - \frac{1}{2}w\left[\frac{1}{2}\left(1-e^{-w^2/4}\right)\right] = \frac{1}{2} - \frac{1}{4}w + \frac{1}{4}we^{-w^2/4}$$

$$u(w,t) = \left(\frac{1}{2} - \frac{1}{4}w + \frac{1}{4}we^{-w^2/4}\right)e^{-\alpha w^2 t}$$

$$u(x,t) = \frac{2}{\pi}\int_2^{\infty} u(w,t)\cos wxdw$$

ج) $0 < m < 1$ $\quad u(x,0) = x^{m-1}$ $\quad u(0,t) = 0$

تبدیل سینوسی $\quad F_s[u_{xx}] = -w^2 u(w,t)$ $\quad u(w,t) = F_s[u(x,t)]$

نسبت به x

$$-\alpha w^2 u(w,t) = u_t(w,t) \to u(w,t) = Ae^{-\alpha w^2 t}$$

$$u(w,0) = A = F_s\left[x^{m-1}\right] = \frac{\tau(m)}{w^m}\sin\frac{m\pi}{2}$$

$$u(w,t) = \frac{\tau(m)}{w^m}\sin\frac{m\pi}{2}e^{-\alpha w^2 t} \to u(x,t) = \frac{2}{\pi}\int_0^{\infty} u(w,t)\sin wxdw$$

د) $u_x(0,t) = 0$ $\quad u(x,0) = x^{m-1}$ $\quad 0 < m < 1$

$u(w,t) = F_c[w(x,t)]$ $\quad F_c[u_{xx}] = -w^2 u(w,t)$

$$-\alpha w^2 u = u_t \rightarrow u(w,t) = Ae^{-\alpha w^2 t}$$

$$u(w,0) = A = F_c\left[x^{m-1}\right] = \frac{\tau(m)}{w^m}\cos\frac{mw}{2}$$

$$u(w,t) = \frac{\tau(w)}{w^m}\cos\frac{mw}{2} \rightarrow u(x,t) = \frac{2}{\pi}\int_0^\infty u(w,t)\cos wx dw$$

هـ) $\quad u_x(0,t) = 0 \qquad u(x,0) = \dfrac{1}{1+x^2}$

$$u(w,t) = Ae^{-\alpha w^2 t} \qquad u(w,0) = A = F_c\left[\frac{1}{1+x^2}\right] = \pi e^{-w} H(x)$$

$$u(w,t) = \pi e^{-|w|} e^{-\alpha w^2 t} \rightarrow u(x,t) = \frac{2}{\pi}\times\pi\int_0^\infty e^{-|w|} e^{-\alpha w^2 t}\cos wx dx$$

هـ – 10 سوال

$$\alpha u_{xx} = u_t \qquad u_x(0,t) = 0 \qquad u(x,0) = \frac{1}{x^2+1}$$

$$u(w,t) = \int_0^\infty u(x,t)\cos wx dx$$

$$\alpha\int_0^\infty u_{xx}\cos wx dx = \int_0^\infty u_t \cos wx dx \quad , \quad \int_0^\infty u\cos wx dx = u(w$$

$$,\int_0^\infty u_{xx}\cos wx dx = I_1 \qquad \begin{cases} u_{xx}dx = dV & ,V = u_x \\ \cos wx = W & ,dW = -w\sin wx \end{cases}$$

$$I = u_x \cos wx \Big]_0^\infty + w\int_0^\infty u_x \sin wx dx \qquad ,\int_0^\infty u_x \sin wx dx = I_2 \begin{cases} u_x dx = dV \\ V = u \\ \sin wx = W \\ dw = w\cos wx \end{cases}$$

$$I_2 = u\sin wx \Big]_0^\infty + w\int_0^\infty u\cos wx dx = -wu(w,t)$$

$$\rightarrow -\alpha w^2 u(w,t) = u_t(w,t)$$

$$\rightarrow u(w,t) = Ae^{-\alpha w^2 t} \qquad u(w,0) = A = \int_0^{\infty} u(x,0)\cos wxdx$$

$$\rightarrow A = \int_0^{\infty} \frac{1}{x^2+1}\cos xdx = ? \qquad F\left(\frac{1}{x^2+1}\right) = ?$$

$$\rightarrow \int_{-\infty}^{+\infty} \frac{1}{x^2+1}(\cos wx - i\sin wx)dx = 2\int_0^{\infty} \frac{1}{x^2+1}\cos wxdx = \pi e^{-|w|}$$

زوج   فرد

$$\rightarrow A = \int_0^{\infty} \frac{1}{x^2+1}\cos wxdx = \frac{\pi}{2}e^{-|w|}$$

$$\rightarrow u(w,t) = \frac{\pi}{2}e^{-|w|}e^{-\alpha w^2 t}$$

$$\rightarrow u(x,t) = \frac{2}{\pi}\int_0^{\infty} u(w,t)\cos wxdxw = \frac{2}{\pi}\int_0^{\infty}\frac{\pi}{2}e^{-|w|}e^{-\alpha w^2 t}\cos xdw$$

سوال 10 – و

$$\alpha u_{xx} = u_t \qquad u(0,t) = 0 \qquad u(x,0) = \frac{x}{(x^2+1)^2}$$

$$u(w,t) = F_s(u(x,t)) = \int_0^{\infty} u(x,t)\sin wxdx$$

$$\alpha\int_0^{\infty} u_{xx}\sin wxdx = \int_0^{\infty} u_t \sin wxdx \qquad ,\int_0^{\infty} u_t\sin wxdx = u_t(w,t)$$

$$\int_0^{\infty} u_{xx}\sin wxdx = u_x\sin wx\Big]_0^{\infty} - w\int_0^{\infty} u_x\cos wxdx$$

I

$$I = \int_0^{\infty} u_x\cos wxdx = u\cos wx + w\int_0^{\infty} u\sin wxdx$$

$$\rightarrow -\alpha w^2 u(w,t) = u_t(w,t) \qquad ,u(w,t) = Ae^{-\alpha w^2 t}$$

$$\rightarrow u(w,0) = A = \int_0^{\infty}\frac{x}{(x^2+1)^2}\sin wxdx = ?$$

$$F\left(\frac{1}{x+1}\right)=\pi e^{-|w|}\rightarrow F\left(\left(\frac{1}{x+1}\right)'\right)=iw\pi e^{-|w|}$$

$$F\left(\frac{x}{(x^2+1)^2}\right)=?\xrightarrow{\text{از قسمت هـ}}$$

$$\rightarrow F\left(\frac{x}{\left(x^2+1\right)^2}\right)=\frac{iw\pi e^{-|w|}}{2}=\int_{-\infty}^{\infty}\frac{x}{(x^2+1)^2}e^{-iwx}dx=\int_{-\infty}^{\infty}\frac{x}{(x^2+1)^2}(\cos wx-i\sin wx)dx$$

فرد   زوج   فرد

$$\rightarrow F\left(\frac{x}{(x^2+1)^2}\right)=-2i\int_0^{\infty}\frac{x}{(x^2+1)^2}\sin wxdx=\frac{iw\pi e^{-|w|}}{2}$$

$$\rightarrow A=\int_0^{\infty}\frac{x}{(x^2+1)^2}\sin xdx=\frac{-w\pi e^{-|w|}}{4}$$

$$\rightarrow u(w,t)=-\frac{w\pi e^{-|w|}}{4}e^{-\alpha w^2 t} \qquad u(x,t)=\frac{2}{\pi}\int_0^{\infty}u(w,t)\sin wxdw$$

$$\rightarrow u(x,t)=\frac{2}{\pi}\int_0^{w}-\frac{w\pi e^{-|w|}}{4}e^{-\alpha w^2 t}\sin wxdw$$

$$a<r<b, 0<\phi<2\pi \qquad u_{rr}+\frac{1}{r}u+\frac{1}{r^2}u_{\phi\phi}=0$$ 11)

الف) $u(a,\phi)=F(\phi)=\phi^2+\cos\phi, u(b,\phi)=g(\phi)=\sin^3\phi$

$$u(r,\phi)=R(r).N(\phi)\rightarrow\frac{r^2R''+rR'}{R}=\frac{-N''}{N}=\lambda$$

→ غیر متناوب

→ غایی $\lambda=0$:

$\lambda=0$: $N(\phi)=A\phi+B$

$$r^2R''+rR'=0\rightarrow R=c+D\ln(r)$$

$$\rightarrow\lambda=0\rightarrow u(r,\phi)=\frac{1}{2}(A+B\ln r)$$

$$\lambda>0:\lambda=\beta^2\rightarrow N''+\beta^2N=0\rightarrow N(\phi)=A\cos\beta\phi+B\sin\beta\phi$$

$$r^2R''+rR'+n^2R\rightarrow R(r)=Cr^n+Dr^{-n}$$

$$u(r,\phi)=\tfrac{1}{2}\left(A_\circ+B_\circ\ln(r)\right)+\sum_{n=1}^{\infty}\left(A_n r^n+B_n r^{-n}\right)\cos(n\phi)+\left(C_n r^n+D_n r^{-n}\right)\sin(n\phi)$$

$$u(a,\phi)=F(\phi)=\tfrac{1}{2}\left(A_\circ+B_\circ\ln(a)\right)+\sum_{n=1}^{\infty}\left(A_n a^n+B_n a^{-n}\right)\cos(n\phi)+\left(C_n a^n+D_n a^{-n}\right)$$

$$\to\sin(n\phi)$$

$$u(b,\phi)=g(\phi)=\tfrac{1}{2}\left(A_\circ+B_\circ\ln(b)\right)+\sum_{n=1}^{\infty}\left(A_n b^n+B_n b^{-n}\right)\cos(n\phi)+\left(C_n b^n+D_n b^{-n}\right)(\sin(n\phi))$$

$$\to B.C:\begin{cases}A_\circ+B_\circ\ln(a)=\tfrac{1}{\pi}\int_0^{2\pi}\left(\phi^2+\cos\phi\right)d\phi=\tfrac{1}{\pi}\left(\tfrac{\phi^3}{3}+\sin\phi\Big|_0^{2\pi}\right)=\dfrac{8\pi^2}{3}\\ A_\circ+B_\circ\ln(b)=\tfrac{1}{\pi}\int_0^{2\pi}\sin^3\phi\,d\phi\left(\sin 3\phi=3\sin\phi-4\sin^3\phi\right)\end{cases}$$

$$=\tfrac{1}{\pi}\left(-\tfrac{3}{4}\cos\phi+\frac{1}{12}\cos 3\phi\Big|_0^{2\pi}\right)=\frac{-3}{4\pi}+\frac{1}{12\pi}+\tfrac{3}{4\pi}-\frac{1}{12\pi}=0$$

معادله را از هم کم می کنیم:

$$B_\circ\left(\ln\left(\tfrac{a}{b}\right)\right)=\frac{8\pi^2}{3}\to B=\tfrac{8\pi^2}{3\ln\left(\tfrac{a}{b}\right)}$$

$$A_\circ=-B_\circ\ln(b)=-\tfrac{8\pi^2}{3\ln(\tfrac{a}{b})}\times\ln(b)$$

$$I:\begin{cases}A_n a^n+B_n a^{-n}=\tfrac{1}{\pi}\int_0^{2\pi}F(\phi)\cos(n\phi)d\phi=\tfrac{4}{n^2}\\ A_n b^n+B_n b^{-n}=\tfrac{1}{\pi}\int_0^{2\pi}g(\phi)\cos(n\phi)d\phi=\begin{cases}0 & n\neq 1,3\\ 0 & n=1\\ 0 & n=3\end{cases}\end{cases}$$

$$II:\begin{cases}C_n a^n+D_n a^{-n}=\tfrac{1}{\pi}\int_0^{2\pi}F(\phi)\sin(n\phi)d\phi=\dfrac{-4\pi}{n}\\ C_n b^n+D_n b^{-n}=\tfrac{1}{\pi}\int_0^{2\pi}g(\phi)\sin(n\phi)d\phi=\tfrac{1}{\pi}\int_0^{2\pi}\sin^3\phi\sin(n\phi)d\phi\end{cases}$$

$$I=\tfrac{1}{\pi}\int_0^{2\pi}\left(\tfrac{3}{4}\sin-\tfrac{1}{4}\sin 3\right)\sin(n\phi)$$

I₁ I₂

$$I_1 = \int_0^{2\pi} \tfrac{3}{4}\sin(n\phi)d\phi = \tfrac{3}{4}\int_0^{2\pi} -\tfrac{1}{2}\left(\cos(n\phi)-\cos(\phi-n\phi)\right)$$

$$= -\tfrac{3}{4}\left[\frac{1}{1+n}\sin(1+n)\phi - \frac{1}{1-n}\sin(\phi-n\phi)\Big|_0^{2\pi}\right] = 0, n=1 \rightarrow \int_0^{2\pi}\tfrac{3}{4}\sin^2\phi = I = \tfrac{3}{4}\pi$$

$$I = -\int_0^{2\pi}\tfrac{1}{4}\sin 3\phi\sin(n\phi)d\phi = +\tfrac{1}{4}\int_0^{2\pi}\tfrac{1}{4}\left(\cos(3\phi+n\phi)-\cos(3\phi-n\phi)\right)$$

$$= \tfrac{1}{8}\left[\frac{1}{3+n}\sin(3+n)\phi - \frac{1}{3-n}\sin(3-n)\phi\Big|_0^{2\pi}\right] = 0, n=3 \rightarrow -\int_0^{2\pi}\tfrac{1}{4}\sin^2 3\phi = -1\tfrac{1}{4}\pi$$

$$I = \begin{cases} 0 & n \neq {}^{+3}_{+1} \\ -\tfrac{1}{4} & n=3 \\ \tfrac{3}{4} & n=1 \end{cases}$$

ب) $u_r(a,\phi) = F(\phi) \qquad , u(b,\phi) = g(\phi)$

از $u(r,\phi)$ نسبت به r مشتق می گیریم و a را جایگذاری می کنیم :

$$u_r(r,\phi) = \frac{1}{2r}B_\circ + \sum_0^\infty\left(nA_n r^{n-1} + (-n)B_n r^{-n-1}\right)\cos(n\phi) + \left(nc_n r^{n-1} - nD_n r^{-n-1}\right)\sin n\phi$$

$$u_r(a,\phi) = \frac{1}{2a}B_\circ + \sum_0^\infty\left(nA_n a^{n-1} - nB_n a^{-n-1}\right)\cos(n\phi) + \left(nC_n a^{n-1} - nD_n a^{-n-1}\right)\sin n\phi = F(\phi)$$

$$u(b,\phi) = \tfrac{1}{2}\left(A_\circ + B_\circ\ln(b)\right) + \sum_0^\infty\left(A_n b^n + B_n b^{-n}\right)\cos n\phi + \left(C_n b^n + D_n b^{-n}\right)\sin n\phi = g(\phi)$$

$$\rightarrow B.C.:\begin{cases} \frac{1}{2a}B_\circ = \tfrac{1}{\pi}\int_0^{2\pi}\left(\phi^2+\cos\phi\right)d\phi = \tfrac{8\pi^2}{3} + B_\circ = 16\pi^2\tfrac{a}{3} \\ A_\circ = -B_\circ\ln(b) = -16\pi^\pi\tfrac{a}{3}\ln(b) \end{cases}$$

$$I:\begin{cases} n(A_n a^{n-1} - B_n a^{n-1}) = \tfrac{4}{\pi^2} \rightarrow \left(A_n a^{n-1} - B_n a^{-n-1}\right) = \frac{4}{n^3} \\ A_n b^n + B_n b^{-n} = 0 \end{cases}$$

$$\rightarrow \left(A_n a^{n-1}b^{-n}\right) + A_n b^n a^{-n-1} = \tfrac{4}{n^3} \rightarrow A_n\left(a^{n-1}b^{-n} + b^n a^{-n-1}\right) = \tfrac{4}{n^3}$$

$$II-:\begin{cases} nc_n a^{n-1} - nD_n a^{-n-1} = -\dfrac{4\pi}{n} \\ C_n b^n + D_n b^{-n} = I = \begin{cases} 0 & n \neq {}^{+3}_{+1} \\ -1/4 & n = 3 \\ 3/4 & n = 1 \end{cases} \end{cases}$$

ج) $u(a,\phi) = F(\phi) \qquad , u_r(b,\phi) = g(\phi)$

$$B.C:\begin{cases} \frac{1}{2}\left(A_\circ + B_\circ \ln(a)\right) = 8\pi^2/3 \\ \frac{1}{2b} B_\circ = 0 \end{cases} \qquad \rightarrow \frac{1}{2} A_\circ = 8\pi^2/3 \rightarrow A_\circ = 16\pi^2/3 , B_\circ = 0$$

$$I:\begin{cases} A_n a^n + B_n a^{-n} = 4/\pi^2 \\ n\left(A_n b^{n-1} - B_n b^{-n-1}\right) = 0 \end{cases}$$

$$II:\begin{cases} C_n a^n + D_n a^{-n} = -\dfrac{4\pi}{n} \\ n\left(C_n b^{n-1} - D_n a^{-n-1}\right) = I \end{cases}$$

د) $u_r(a,\phi) = f(\phi), u_r(b,\phi) = g(\phi)$

$$B.C:\begin{cases} \dfrac{1}{2a} B_0 = 16\pi^2 a/3 \\ \dfrac{1}{2a} B_0 = 0 \end{cases}$$

$$I:\begin{cases} n(A_n a^{n-1} - B_n a^{n-1}) = 4/n^2 \\ n(A_n b^{n-1} - B_n b^{-n-1}) = 0 \end{cases} \longrightarrow A_n a^{n-1} b^{-n-1} + A_n b^{n-1} a^{-n-1} = 4/n^3$$

$$II:\begin{cases} n(C_n a^{n-1} - B_n a^{-n-1}) = \dfrac{-4\pi}{n} \\ n(C_n b^{n-1} - B_n b^{-n-1}) = I \end{cases}$$

سوال 12

$$u_{rr} + \frac{2}{r}u_r + \frac{1}{r\sin\theta}\frac{5}{5\theta}\left(\sin\theta\frac{5u}{5\theta}\right) = 0 \qquad u(R,\theta) = F(\theta) \qquad u(R,\theta) = M(R)N(\theta)$$

$$M''N + \frac{2}{R}M'N + \frac{1}{r^2 Sin\theta}\frac{d}{d\theta}(Sin\theta MN'') = 0 \rightarrow \frac{M''}{M} + \frac{2}{R}\frac{M'}{M} + \frac{1}{Nr^2 Sin\theta}(N''Sin\theta)(siNn\theta) = 0$$

$$r^2\frac{M''}{M} + 2r\frac{M'}{M} = -\frac{1}{NSin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dN}{d\theta}\right) = \lambda$$

$r^2M''+2rM'-\lambda M = 0 \rightarrow$ معادله اولدکشی $\quad K^2 + K - \lambda = 0$

$$K_1 = \frac{-1+\sqrt{1+4\lambda}}{2}$$

$$K_2 = \frac{-1-\sqrt{1+4\lambda}}{2}$$

$$K_1 = \alpha = -\frac{1}{2} + \sqrt{\lambda + \frac{1}{4}} \rightarrow -\alpha = \frac{1}{2} - \sqrt{\lambda + \frac{1}{4}} \rightarrow -\alpha - 1 = -\frac{1}{2} - \sqrt{\lambda + \frac{1}{4}}$$

$$K_1K_2 = -\lambda = -\alpha(\alpha+1) \qquad M(r) = C_1 r^{\alpha} + C_2 r^{-(\alpha+1)}$$

$$\frac{d}{d\theta}\left(\sin\frac{dN}{d\theta}\right) + \lambda N\sin\theta + 0 \qquad \hat\lambda = r\cos\theta \xrightarrow{r=1} \hat\lambda = \cos\theta$$

$$\frac{dN}{d\theta} = \frac{dN}{d\hat\lambda}\cdot\frac{d\hat\lambda}{d\theta} = -\sin\theta\frac{dN}{d\hat\lambda} \rightarrow -\sin\theta\frac{d}{d\hat\lambda}\left[\sin(-\sin\theta)\frac{dN}{d\hat\lambda}\right] + \alpha(\alpha+1)N\sin\theta = 0$$

$$-\frac{d}{d\hat\lambda}\left[-\sin^2\theta\frac{dN}{d\hat\lambda}\right] + \alpha(\alpha+1)N = 0 \rightarrow \frac{d}{d\hat\lambda}\left[(1-\hat\lambda^2)\frac{dN}{d\hat\lambda}\right] + \alpha(\alpha+1)N = 0$$

$$\begin{cases} p_n(\hat\lambda) = \dfrac{1}{2^n ni}\dfrac{d^n}{d\hat\lambda^n}\left[(\hat\lambda-1)^2\right] \\ Q_n(\hat\lambda) = p_n(\hat\lambda)\displaystyle\int\frac{d\hat\lambda}{(1-\hat\lambda^2)p_n{}^2(\hat\lambda)} \end{cases}$$ رودریگز

$$u(r,\theta) = \left[C_1r^n + C_2r^{-(n+1)}\right]\left[B_2p_n(\cos\theta) + B_2Q_n(\cos\theta)\right]$$

الف)

$$u(a,\theta)= f(\theta)= f\left(\cos^{-1}\mu\right)= f(\mu)=\sum_{n=1}^{\infty}\left[A_n a^n + B_n a^{-(n+1)}\right]p_n(\cos\theta)$$

$$u(,b\vartheta)= g(\vartheta)= g\left(\cos^{-1}\mu\right)= G(\mu)=\sum_{n=1}^{\infty}\left[A_n b^n + B_n b^{-(n+1)}\right]p_n(\mu)$$

$$\int_{-1}^{1}F(\vartheta)p_m(\cos\theta)\sin\theta d\theta=\int_{-1}^{1}\sum_{n=0}^{\infty}\left[A_n a^n + B_n a^{-(n+1)}\right]p_n(\cos\theta)p_m(\cos\theta)\sin\theta d\theta$$

$$\int_{-1}^{1}f(\theta)p_m(\cos\theta)\sin\theta d\theta=\left[A_n a^n + B_n a^{-(n+1)}\right]\times\frac{2}{2n+1}$$

$$\left[A_n a^n + B_n a^{-(n+1)}\right]=\frac{2n+1}{2}\int_{-1}^{1}f(\vartheta)p_n(\cos\theta)\sin\theta d\theta$$

$$u(r,\theta)=\sum_{n=1}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]P_n(Cos\theta)$$

قسمت ب)

وقتی r 0 به سمت ∞ میل می کند

$B_n r-(n+1)$

$$u(r,\theta)=\sum_{n=1}^{\infty}\left[A_n r^n\right]P_n(Cos\theta)$$

$$u(a,\theta)= f(\theta)=\sum_{n=1}^{\infty}\left[A_n a^n\right]P_n(Cos\theta)$$

$$\int_{-1}^{1}f(\theta)P(Cos\theta)Sin\theta d\theta=\int_{-1}^{1}\sum_{n=1}^{\infty}A_n a^n P_m(Cos\theta)P_n(Cos\theta)Sin\theta d\theta$$

$$\int_{-1}^{1}f(\theta)P_m(Cos\theta)Sin\theta d\theta = A_n a^n\,\frac{2}{2n+1}$$

$$A_n=\frac{2n+1}{2a^n}\int_{-1}^{1}f(\theta)P_m(Cos\theta)Sin\theta d\theta$$

$$A_n=\frac{2n+1}{2a}\int_{-1}^{1}(\theta+Cos^2\theta)P_n(Cos\theta)Sin\theta d\theta$$

$$P_0(Cos\theta)=1$$
$$P_1(Cos\theta)=Cos\theta$$
$$P_2(Cos\theta)=\frac{1}{2}(3Cos^2\theta-1)$$
$$P_3(Cos\theta)=\frac{1}{2}\left(5Cos^3\theta-3Cos\theta\right)$$

$$A_0=\frac{1}{2}\int_{-1}^{1}(\theta+Cos^2\theta)P_0(Cos\theta)Sin\theta d\theta$$

فـرمـول رودریـگز

ج)

$$u(r,\theta)=\sum_{n=0}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]\left[p(\cos\theta)\right]$$

$A_n r^n$ وقـتـی $r\to\infty$ بـه سمت ∞ مـیـل مـی کـنـد $A_n=0\leftarrow$

$$u(r,\theta)=\sum_{n=0}^{\infty}\left[B_n r^{-(n+1)}\right]\left[P_n\cos\theta\right]$$

$$u(a,\theta)=F(\vartheta)=\sum_{n=0}^{\infty}\left[B_n a^{-(n+1)}\right]P_n(\cos\theta)$$

$$\int_{-1}^{+1}F(\theta)P_m(\cos\theta)\sin\theta d\theta=\int_{-1}^{1}\sum_{n=0}^{\infty}B_n a^{-(n+1)}P_n(\cos\theta)P_m(\cos\theta)\sin\vartheta d\theta$$

$$\int_{-1}^{1}F(\theta)P_m(\cos\vartheta)\sin\theta d\theta=\frac{2}{2n+1}\frac{B_n}{a^{n+1}}$$

$$B_n=\frac{(2n-1)a^{n+1}}{2}\int_{-1}^{1}F(\vartheta)P_m(\cos\vartheta)\sin\theta d\theta$$

سوال 13

$\nabla^2 u=0 \qquad 0<y<a \qquad ,x>0$

الف) $u(x,0)=u(x,a)=0 \qquad u(0,y)=e^{-y}\sin y$

معادلـه خطی و همگن مـی بـاشد و شـرایـط مـرزی در جهت y همگن است.

$u(x,y)=M(x)N(y)$

$$\frac{M''(x)}{M}+\frac{N''(y)}{N}=0$$

وابـسته بـه y

$$\frac{M''(x)}{M}=-\frac{N''(y)}{N}=\lambda\to\begin{cases}M''(x)-\lambda M=0\\ N''(y)+\lambda N=0\end{cases}$$

وابـسته بـه x